A Mechanism Design Approach to Government Funding Allocation in Pharmaceutical R&D: Theory and Experiments

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Problem definition: Governments actively provide funding to support pharmaceutical R&D, especially at early stages. Beyond maximizing expected payoff, public funders have broader responsibilities, including obtaining accurate information from firms, such as private R&D costs, to support the design of effective subsidies and policy interventions. However, firms may strategically misreport these costs when competing for public funds. This creates a dual challenge: allocating limited budgets efficiently while also eliciting accurate information from firms. We therefore seek to design a funding-allocation mechanism that induces truthful revelation of private cost information and theoretically maximizes the government's expected payoff, while also performing well in practice.

Methodology/results: We study a static setting with multiple firms and a budget-constrained government funder, and characterize the incentive-compatible optimal allocation mechanism. We then conduct laboratory experiments comparing our mechanism with two benchmarks: a threshold mechanism and a first-price-auction-based (FPA-based) mechanism. The results show that the proposed mechanism reliably elicits truthful cost information across budget regimes and that its empirical performance aligns closely with theoretical predictions. By contrast, although the FPA-based mechanism performs similarly to the optimal mechanism in terms of payoff empirically, it generates systematically distorted reports due to risk aversion and ambiguity aversion, thereby undermining the funder's ability to learn firms' true cost information.

Managerial implications: Supported by both theoretical and experimental evidence, our proposed mechanism provides guidance for designing public funding policies in pharmaceutical R&D that promote truthful information revelation while supporting broader public-sector and public-health objectives.

Key words: mechanism design, lab experiment, behavioral operations, resource allocation, pharmaceutical R&D

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1. Introduction

Pharmaceutical R&D plays a central role in improving public health, yet financial frictions distort which drug-development projects are pursued, with firms cutting back especially on novel and high-uncertainty candidates (Krieger et al. 2022a). To address these frictions, governments and other public-sector organizations provide funding to support pharmaceutical R&D. A central challenge in designing such funding programs is information asymmetry: firms hold private information about their development costs, and public funders may have difficulty verifying this information when allocating funds (Baron and Myerson 1982, Laffont and Tirole 1993). Without credible information revelation, the funder may misjudge which projects to support and how much funding to allocate (Wright 1983, Kremer 1998, Kremer and Glennerster 2004). Beyond the allocation decision, the cost data elicited from each cohort of funded firms can have broader value for the agency's long-term policy learning, for example, by helping it update its beliefs about the industry's cost distribution and refine future programs.

This paper studies the optimal design of such funding mechanisms in a setting motivated by U.S. government programs for early-stage drug development, most directly the NIH SBIR/STTR awards, the largest non-dilutive federal funding channel for small-business biomedical innovation. We design an allocation mechanism that maximizes the government agency's expected payoff. By the revelation principle (Myerson 1981), we restrict attention to incentive-compatible direct mechanisms without loss of generality. Truthful reporting is therefore an equilibrium, and the government agency obtains accurate cost information to guide both funding decisions and broader policy objectives. Three features of the institutional setting shape our modeling choices. First, public support typically targets pre-commercial development, where projects generate little or no revenue and firms rely heavily on external funding to finance development costs. Second, many funded firms are small biotechs or startups with limited track records, which plausibly have better information about their own development costs (e.g., research capacity, personnel, and input requirements) than about technical success probabilities, which are often difficult to assess reliably at the preclinical or Phase I stage. We therefore model firms' private information as cost rather than success probability. Third, public funders operate under hard budget caps reflecting legislative appropriations, which we incorporate as an ex-post budget constraint.

While our theoretical mechanism assumes fully rational, payoff-maximizing agents, a substantial experimental literature documents that subjects systematically deviate from truthful reporting in incentive-compatible mechanisms, particularly when the strategic environment is complex (Kagel and Levin 1993, Chen and Sönmez 2006, Li 2017). Because such deviations could undermine both the incentive-compatibility and the expected performance of our mechanism, we complement the theoretical analysis with laboratory experiments designed to test whether subjects in fact report truthfully and to identify any systematic deviations that may arise.

To guide the empirical analysis, we propose two hypotheses regarding the mechanism's performance. First, we hypothesize that under our mechanism, participants will report their costs truthfully (Hypothesis 1). Second, we hypothesize that the optimal mechanism outperforms the benchmark mechanisms in expected payoff (Hypothesis 2). To test these hypotheses, we evaluate our optimal mechanism against two commonly proposed benchmarks for R&D funding allocation: a threshold mechanism representing cost-based screening, in which the funder allocates equal investment to all projects with reported costs below a fixed threshold, and a first-price-auction-based (FPA-based) mechanism representing competitive bidding, in which the funder allocates funding to the firms with the lowest reported costs in order until the budget is exhausted.

Our laboratory experiment follows a 2×3 experiment design, with mechanism condition (our proposed mechanism versus benchmark mechanisms) and budget level (low, medium, or high). Participants act as firms in a simulated pharmaceutical R&D funding environment and strategically report their private costs when interacting with a government funder. We recruited 396 participants through Amazon Mechanical Turk (MTurk). To ensure data quality and engagement, we included an attention check to exclude inattentive participants and implemented an incentive scheme that awarded bonuses to participants who performed better than their peers.

In line with Hypothesis 1, we find that under the optimal mechanism, reported costs do not differ significantly from true private costs. Moreover, relative to both the threshold and FPA-based benchmark mechanisms, the optimal mechanism yields smaller reporting deviations, indicating that it is more effective at inducing truthful reporting. These findings provide strong support for the mechanism's objective of eliciting truthful private cost information. By contrast, the payoff results are more mixed: although the optimal mechanism performs largely in line with theoretical predictions, it does not significantly outperform the FPA-based mechanism across budget conditions, because distorted reporting under the FPA-based mechanism improves that mechanism's payoff performance, thereby failing to support Hypothesis 2.

Further analysis suggests that this discrepancy arises primarily from the FPA-based mechanism rather than from failures of the optimal mechanism itself. In particular, agents under the FPA-based mechanism appear to adopt overly conservative reporting strategies. To further investigate the behavioral explanation for this reporting pattern, we conduct a second laboratory experiment and find that risk aversion (Pratt 1978) and ambiguity aversion (Ellsberg 1961) help explain this behavior and, in turn, enhance the payoff performance of the FPA-based mechanism, albeit at the cost of substantially distorted cost reports.

Our study makes two main contributions. First, we characterize the optimal mechanism for a budget-constrained funder allocating early-stage R&D support to firms with privately known costs, in a setting calibrated to the institutional features of U.S. government drug-development programs. Second, we use laboratory experiments to study behavioral deviations from rationality in the funding-allocation process, identify behavioral explanations for those deviations, and assess the practical performance of the mechanism. To the best of our knowledge, this study is among the first to combine theoretical analysis and experimental evidence to study truthful information elicitation in government funding allocation for pharmaceutical R&D. The information elicited through such a mechanism can support not only the allocation decision at hand but also broader policy objectives, including future subsidy design, project prioritization, and related public-sector interventions.

The remainder of the paper is organized as follows: §2 summarizes the related literature; §3 discusses the problem and introduces the model; §4 characterizes the optimal funding mechanism; §5 develops the hypotheses and describes the laboratory experiments; §6 presents the empirical results and discusses behavioral deviations from rationality; and §7 concludes.

2. Literature Review

Our paper is related to the existing literature on (1) resource allocation in the public sectors, (2) pharmaceutical R&D, (3) mechanism design with budget constraints, and (4) behavioral operations. In this section, we review the relevant literature and outline the contributions of our work.

Resource allocation in the public sectors. The resource allocation problem in the public sector has been extensively studied in prior literature. Because public agencies carry significant social responsibilities, revenue maximization is not their primary objective when allocating limited resources. Instead, they focus on promoting social welfare, such as reducing wealth inequality, providing fundamental support for disadvantaged populations and resource-limited regions, and fostering societal business innovation. For example, Tingley and Liebman (1984) formulates a linear integer optimization problem to support state-level resource allocation in the U.S. Department of Agriculture's Special Supplemental Nutrition Program for Women, Infants, and Children (WIC). Lien et al. (2014) examines the problem of nonprofit organizations distributing a scarce resource to meet customer demands that arrive sequentially, using a dynamic programming framework. Roet-Green and Shetty (2022) studies the resource allocation problem faced by a welfare-maximizing service provider who must distribute a fixed quantity of resources between two service variants, a standard option and an expedited option, with an application to U.S. airports through the TSA PreCheck program. Singh and Wu (2025) explores the efficient allocation of a divisible resource, such as water in water-scarce regions, and proposes mechanisms to maximize aggregate consumer welfare, accounting for consumers' heterogeneous incomes and private valuations of the resource.

In the context of R&D, prior studies have examined how public sector institutions can enhance R&D efficiency and promote societal business innovation. For instance, Bruce et al. (2019) studies U.S. federal R&D contracts, which fall into two categories: grants, which involve minimal oversight, and cooperative agreements, which grant decision rights during the project. They show that cooperative agreements can improve efficiency for early-stage, high-uncertainty projects, particularly when government scientists with relevant expertise are located near the firm's R&D site. Similarly, Gao et al. (2022) explores optimal resource allocation strategies for both nonprofit and profit-maximizing principals who must allocate limited resources to support innovations by multiple, potentially competing, innovators. A recurring challenge in R&D funding allocation is the presence of private information held by agents, which can undermine allocation efficiency and ultimately harm social welfare (Esteban and Ray 2006, Singh and Wu 2025). To address this issue, recent research highlights the importance of designing mechanisms that incentivize agents to truthfully report their private information. Truthful reporting enables better decision-making and can lead to improved social outcomes, particularly in public sector R&D efforts. Our study also highlights the importance of designing mechanisms that incentivize agents to truthfully report their private information in the pharmaceutical R&D setting.

Pharmaceutical R&D. While general R&D allocation studies provide broad frameworks applicable across industries, a growing stream of research focuses specifically on pharmaceutical R&D, addressing the unique challenges of drug development and offering more industry-specific insights. For example, Vernon (2005) studies the adverse impact of price regulation on pharmaceutical R&D investment. Chan et al. (2007) develops a dynamic programming model to explain why firms adopt time-varying strategic thresholds when selecting R&D projects. Ganuza et al. (2009) analyzes the incentives for pursuing minor improvements to existing compounds and finds that small innovations often receive disproportionately high rewards, as firms tend to target inelastic segments of demand. Rao (2020) builds a dynamic investment model to analyze the strategic decisions of a pharmaceutical firm in response to its competitors in drug R&D, and estimates structural parameters using data from phase III trials. Krieger et al. (2022b) investigates how firms' investment decisions are affected by negative shocks to existing products, using FDA public health advisories as exogenous shocks to approved drugs. Our study contributes to this stream of literature by proposing a theoretical mechanism that considers the characteristics of pharmaceutical R&D and by validating its practical performance through experimental studies. It aims both to elicit truthful private cost information from firms and to maximize the expected payoff of governments.

Mechanism design with budget constraints. Our theoretical analysis builds on the literature on auctions with financial constraints, in which a principal allocates scarce resources among privately informed, budget-constrained agents. Beginning with Laffont and Robert (1996), this literature has examined how budget constraints distort optimal and efficient allocation under various information structures: commonly known versus private budgets (Maskin 2000, Che and Gale 2000, Pai and Vohra 2014), symmetric versus asymmetric bidders (Malakhov and Vohra 2008), and single- versus multi-object settings (Benoit and Krishna 2001). The common feature is a buyer-side budget, i.e., each bidder's payment cannot exceed an exogenous or privately known liquidity limit. Key distinctions in our investment problem are that the sum of monetary transfers to agents is bounded, rather than the charge to each bidder as in these problems, and that the quantity of the good being sold is not restricted to one. In other words, the funder can allocate resources to multiple firms, with each share not exceeding one. These features make our problem mathematically distinct.

Behavioral operations. Finally, our study is closely related to the literature on behavioral biases in decision-making. Theoretical analyses, especially those based on utility-maximizing models, typically assume rational behavior. In practice, however, decision makers often deviate from normative predictions, and such deviations can reduce the performance of theoretically optimal policies (Donohue et al. 2019). For this reason, an increasing number of studies examine behavioral biases in operational decision-making. For example, Beer et al. (2026) show that workers often deviate from the optimal policy by taking longer than normative theory prescribes to complete their tasks or by failing to complete the project. In a related setting, Snyder et al. (2026) show that workers' use of algorithmic advice depends on system load and algorithm quality, and that greater reliance on algorithms does not necessarily translate into faster decisions. Long et al. (2020) experimentally study continue/abandon decisions in multi-stage projects and show that project termination is often delayed, with the observed patterns being explained by behavioral factors such as sunk cost bias, status quo bias, and reference dependence. Davis et al. (2022) develop a normative model of procurement under asymmetric information and then use a human-subjects experiment to test the model's predictions, documenting both broad qualitative support and managerially relevant deviations. Beil et al. (2025) study human decision-making in dynamic resource allocation and show that subjects perform substantially worse as the problem becomes more complex. Their analysis suggests that such deviations are associated with heuristics such as mis-weighting future periods and focusing excessively on the highest-value opportunities. We contribute to this literature by experimentally examining behavioral deviations under a designed funding-allocation mechanism. Our study provides implications for government investment by testing whether a theoretically optimal mechanism effectively motivates agents to reveal truthful private information in practice, thereby helping policymakers allocate resources more effectively and design funding policies that better align private incentives with public goals.

3. Model

Consider a setting in which a government agency (the principal, hereafter the "funder") interacts with $N \geq 2$ agents, indexed by $i \in [N] \equiv \{1, \ldots, N\}$, in a static game. Each agent represents a firm undertaking an early-stage drug development project, for example, a drug candidate at the preclinical or Phase I stage, targeting a disease of public-health importance.[1] Such projects are often supported through public programs for early-stage pharmaceutical innovation, including NIH small-business programs such as SBIR/STTR. All participants are risk-neutral. Each project, if funded, yields an expected societal value of R to the funder, where R can be interpreted as the product of the project's probability of successful launch and its social value conditional on success. Firm i's development cost, denoted by c_i, is independently and identically drawn from a distribution with support $[c_L, c_H]$, cumulative distribution function F, and density f. The realization of c_i is privately known to firm i.

We model a firm's private information as its development cost rather than its probability of success because, at this early stage, startups and small biotech firms typically lack reliable estimates of their success probabilities owing to limited track records and high technical uncertainty, whereas they have better information about their own research capacity, personnel, and input requirements. Since the funder optimally never funds firms with $c_i > R$, we assume $c_H \leq R$ without loss of optimality. The funder has a limited budget B, reflecting a legislative appropriation or program-level funding cap, and aims to maximize its expected payoff, defined as the expected social value of funded projects net of total transfers.

By the revelation principle (Myerson 1981), we can restrict attention to direct symmetric incentive-compatible mechanisms.[2] That is, given the reported cost vector $\mathbf{c} \in [c_L, c_H]^N$, the mechanism specifies: (1) an allocation rule $q(c_i, c_{-i})$, denoting the share of firm i's project value claimed by the funder; (2) a monetary transfer $m(c_i, c_{-i})$ from the funder to firm i. The variable q abstracts from the specific contractual form through which the funder's claim is implemented, for instance, a royalty entitlement, a government-use license, or a priority purchase right. Similarly, the transfer m encompasses the diverse financial vehicles used to subsidize the firm, such as upfront grants or milestone-based payments. Here, c_i denotes firm i's reported type, and $c_{-i} \equiv (c_1, c_2, \ldots, c_{i-1}, c_{i+1}, \ldots, c_N)$ denotes the vector of reported types of all other firms.

To complete the description of the environment, we now specify the firm's payoff outside the funder's mechanism. After accepting the funder's contract, the firm may sell its retained share $(1 - q)$ of the project to outside investors, for example, downstream pharmaceutical partners or

[1] Although we motivate the analysis through drug development, the model applies more broadly to government funding of early-stage, pre-revenue projects with socially valuable outputs and privately known development costs.

[2] The symmetry follows from footnote 11 in Maskin and Riley (1984).

licensees, who provide upfront capital in exchange for a contingent claim on the project's future commercial value. For the early-stage projects we study, the social value R accruing to the funder incorporates public-health benefits (health externalities, insurance value against future disease burden, and welfare gains to populations with limited ability to pay commercial prices) that private investors capture only partially through product prices or licensing revenue. This wedge between social and private returns is precisely the economic rationale for public R&D funding in this setting: if private markets could fully internalize R, the funder would have no role to play. Consistent with this public-goods interpretation, we assume that the firm's net payoff from selling the retained share to outside investors is zero, with their upfront contribution covering the firm's development cost on the retained share. Since early-stage drug candidates also generate no product revenue prior to regulatory approval, the firm's only source of positive payoff is the transfer received from the government funder.

Let $\hat{u}(c, \hat{c})$ denote firm i's *expected* payoff when its true cost is c but it reports $\hat{c}$. By symmetry, this function is identical across firms, so we suppress the firm index. Upon observing the reported types of the other firms, $c_{-i} \in [c_L, c_H]^{N-1}$, the mechanism specifies a monetary transfer $m(\hat{c}, c_{-i})$ to firm i and a share $q(\hat{c}, c_{-i})$ of the project claimed by the funder, imposing an expected cost of $c\,q(\hat{c}, c_{-i})$ on the firm. The firm's expected payoff is therefore the monetary transfer minus its share of the development cost, i.e.,

$$\hat{u}(c, \hat{c}) = \int_{[c_L, c_H]^{N-1}} [m(\hat{c}, c_{-i}) - c\,q(\hat{c}, c_{-i})] \prod_{k \in [N] \setminus \{i\}} \mathrm{d}F(c_k), \quad \forall (c, \hat{c}) \in [c_L, c_H]^2. \tag{1}$$

The funder's expected payoff under truthful reporting is

$$U \equiv N \int_{[c_L, c_H]^N} [R\,q(c_1, c_{-1}) - m(c_1, c_{-1})] \prod_{k=1}^{N} \mathrm{d}F(c_k). \tag{2}$$

A feasible mechanism must satisfy the incentive-compatibility constraint, meaning that a firm with cost c should not benefit from misreporting its type as $\hat{c}$:

$$\hat{u}(c, c) \geq \hat{u}(c, \hat{c}), \quad \forall (c, \hat{c}) \in [c_L, c_H]^2. \tag{IC}$$

Furthermore, given the outside-option assumption established in the preceding paragraph, the following individual rationality constraint must hold to guarantee the firm's participation:

$$\hat{u}(c, c) \geq 0, \quad \forall c \in [c_L, c_H]. \tag{IR}$$

In our setting, the truthful reporting induced by the (IC) constraint carries institutional weight beyond its standard theoretical purpose. While IC follows from the revelation principle, the elicited cost data have substantial practical value: government funders are repeat players who use cost data from each round to inform future programs, i.e., updating beliefs about F and informing the design of follow-on rounds. Because each round funds a largely distinct cohort of firms, this cross-period informational value enters as a funder-side consideration rather than a strategic concern for the firms themselves, leaving the static framework appropriate for the screening problem we study.

Beyond the standard (IC) and (IR) constraints, the mechanism must satisfy three additional feasibility conditions. First, the funder's limited budget B caps the total transfer; that is,

$$\sum_{i=1}^{N} m(c_i, c_{-i}) \leq B, \quad \forall \mathbf{c} \in [c_L, c_H]^N. \tag{3}$$

This is an ex-post constraint, reflecting that congressional appropriations cannot be exceeded in any state of the world. Second, the share claimed by the funder must lie in $[0,1]$:

$$0 \leq q(c_i, c_{-i}) \leq 1, \quad \forall \mathbf{c} \in [c_L, c_H]^N. \tag{4}$$

Finally, transfers must be non-negative, imposing a limited-liability constraint that rules out the funder charging the firm:

$$m(c_i, c_{-i}) \geq 0, \quad \forall \mathbf{c} \in [c_L, c_H]^N. \tag{5}$$

To summarize, the funder's problem can be formulated as follows:

$$\max_{\{q(\cdot), m(\cdot)\} \in \Omega_0} U, \tag{6}$$

where the feasible set Ω_0 is determined by linear constraints (IC), (IR), (3), (4), and (5).

The optimization problem (6) involves two N-dimensional functions, making it technically challenging to solve directly. To simplify the analysis, we introduce the following *interim* allocation and payment functions

$$Q(c_i) \equiv \mathbb{E}_{c_{-i}}[q(c_i, c_{-i})] = \int_{[c_L, c_H]^{N-1}} q(c_i, c_{-i}) \prod_{k \in [N] \setminus \{i\}} \mathrm{d}F(c_k), \quad \forall c_i \in [c_L, c_H] \tag{7}$$

and

$$M(c_i) \equiv \mathbb{E}_{c_{-i}}[m(c_i, c_{-i})] = \int_{[c_L, c_H]^{N-1}} m(c_i, c_{-i}) \prod_{k \in [N] \setminus \{i\}} \mathrm{d}F(c_k), \quad \forall c_i \in [c_L, c_H]. \tag{8}$$

The functions Q and M represent the firm's expected share claimed by, and expected payment received from, the funder, respectively, when it reports its cost as c_i, assuming all other firms report their private information truthfully. Both parties' payoff functions, defined by (1) and (2), can be rewritten as

$$\hat{u}(c, \hat{c}) = M(\hat{c}) - c\,Q(\hat{c}), \quad \forall (c, \hat{c}) \in [c_L, c_H]^2$$

and

$$U = N \int_{c_L}^{c_H} [R\,Q(c) - M(c)]\,\mathrm{d}F(c).$$

Border (1991) proves that the budget constraint (3) holds if and only if[3]

$$\int_{c_L}^{c} M(y)\,\mathrm{d}F(y) \le \frac{B}{N}\left[1-[1-F(c)]^N\right], \quad \forall c \in [c_L, c_H]. \tag{9}$$

The remaining feasibility constraints (4) and (5) boil down to

$$0 \le Q(c) \le 1, \quad \forall c \in [c_L, c_H], \text{ and} \tag{10}$$

$$M(c) \ge 0, \quad \forall c \in [c_L, c_H], \tag{11}$$

respectively. The funder's problem can be reformulated as follows:

$$\max_{\{Q(\cdot), M(\cdot)\} \in \Omega} U, \tag{12}$$

where the feasible set Ω is determined by linear constraints (IC), (IR), (9), (10), and (11).

4. Optimal Mechanism

In this section, we characterize the optimal mechanism that solves the funder's problem (12) and the corresponding implementation. We make the following assumption throughout the paper.

ASSUMPTION 1. *The density function f is non-increasing.*

Assumption 1 is standard in the mechanism design literature. It implies that the inverse hazard ratio, $F(c)/f(c)$, is increasing, and it holds for several commonly used distributions, including the uniform, truncated exponential, and Pareto distributions.

Let

$$w(c) \equiv R - c - \frac{F(c)}{f(c)}, \quad \forall c \in [c_L, c_H], \tag{13}$$

denote the virtual valuation function. The structure of the optimal mechanism is highly sensitive to the funder's budget B. Proposition 1 analyzes the case in which the funder's budget is limited. We defer all proofs to Appendix EC.4.

PROPOSITION 1. *Define*

$$h(c) \equiv -c\,w(c)\,f(c) + \int_{c_L}^{c} w(y)\,f(y)\,\mathrm{d}y, \quad \forall c \in [c_L, c_H]. \tag{14}$$

When $B \le \underline{B}$, the optimal reduced-form mechanism is given by

$$Q(c) = \begin{cases} B\left[\frac{[1-F(c)]^{N-1}}{c} - \int_c^{\tilde{c}} \frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y\right], & \forall c \in [c_L, \tilde{c}], \\ 0, & \forall c \in (\tilde{c}, c_H], \end{cases} \qquad M(c) = \begin{cases} B\,[1-F(c)]^{N-1}, & \forall c \in [c_L, \tilde{c}], \\ 0, & \forall c \in (\tilde{c}, c_H], \end{cases} \tag{15}$$

[3] This reformulation requires the monotonicity of the interim allocation rule Q, which is standard in mechanism design problems (see Lemma EC.1).

where the thresholds $\tilde{c}$ *and* $\underline{B}$ *are defined by*

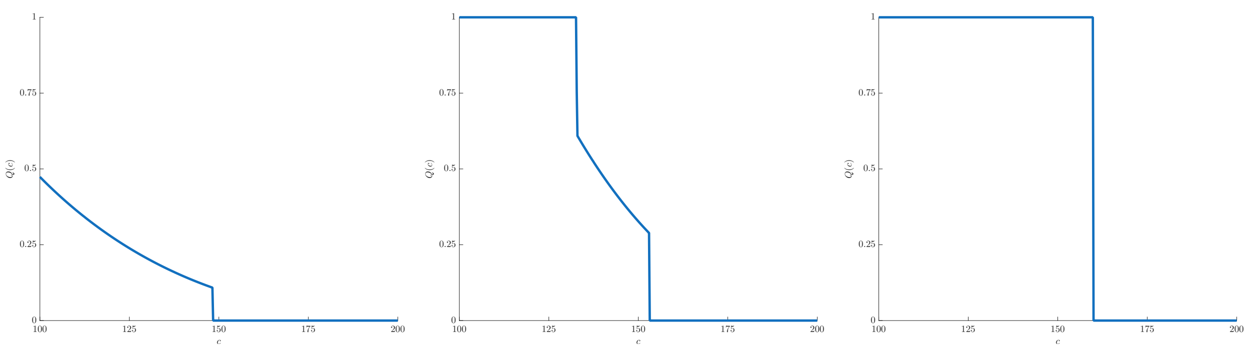
$$\tilde{c} \equiv \sup\{c \in [c_L, c_H] : h(c) \leq 0\} \tag{16}$$

and

$$\underline{B} \equiv \left[\frac{1}{c_L} - \int_{c_L}^{\tilde{c}} \frac{[1-F(y)]^{N-1}}{y^2} \mathrm{d}y\right]^{-1}, \tag{17}$$

respectively.

Figure 1 **The optimal interim allocation rule Q specified in Propositions 1, 2, and 4, respectively. The model parameters are $c_L = 100$, $c_H = 200$, $R = 220$, $f(c) = 1/100$ for all $c \in [c_L, c_H]$, and $N = 3$.**[4]



(a) $B = 60$ (b) $B = 200$ (c) $B = 500$

In this scenario, the funder's budget is limited, and the interim allocation rule Q remains strictly below 1 (see Figure 1(a)). This implies that the funder does not claim the entire project, even for firms with the lowest cost (or, equivalently, the highest return). The following corollary provides an implementation rule for the reduced-form mechanism defined in (15).

Corollary 1. *The reduced-form mechanism defined by* (15) *can be implemented as follows: for any* $(c_i, c_{-i}) \in [c_L, c_H]^N$,

$$q(c_i, c_{-i}) = Q(c_i), \quad m(c_i, c_{-i}) = \begin{cases} B, & \textit{if } c_i < \min_{j \in [N] \setminus \{i\}} c_j \textit{ and } c_i \leq \tilde{c}, \\ 0, & \textit{otherwise.} \end{cases} \tag{18}$$

Since the probability that two or more firms report the same minimal cost is zero, without loss of generality, we omit a tie-breaking rule and allocate no funding in such cases. This convention applies throughout the remainder of this section.

The implementation rule proposed in Corollary 1 is as follows: if a firm reports a cost c below the threshold $\tilde{c}$, the funder claims a share $Q(c)$ from it, where $Q(\cdot)$ is a decreasing function. However,

[4] Given the model parameters, which are used in the experimental setting in Sections 5 and 6, the optimality conditions in Proposition 3 are never satisfied, and thus this case does not arise.

a monetary transfer occurs only if the firm reports the lowest cost among all participants and that cost is below $\tilde{c}$. In that case, the funder allocates the entire budget B to the firm. Figure A1(a) in Appendix EC.3 visualizes the implementation rule when $N = 2$.

As the funder's budget increases, it claims the entire project from firms with sufficiently low costs. However, for firms with higher costs, the funder continues to claim a share strictly less than 1. This behavior is formally characterized in the following proposition.

Proposition 2. *Define*

$$c_0 \equiv \sup\{c \in [c_L, c_H] : w(c) \geq 0\}. \tag{19}$$

Recall that h, $\tilde{c}$, *and* $\underline{B}$ *are defined in* (14), (16), *and* (17), *respectively. When* $\tilde{c} < c_H$, $B > \underline{B}$, *and one of the following conditions holds:*

1. $c_0 < c_H$ *and*

$$B < \bar{B} \equiv \frac{N\, c_0\, F(c_0)}{1 - [1 - F(c_0)]^N}; \text{ or} \tag{20}$$

2. $c_0 = c_H$ *and*

$$B < \tilde{B} \equiv \frac{\tilde{c}_1\, F(\tilde{c}_1)}{\frac{1}{N}\left[1 - [1 - F(\tilde{c}_1)]^N\right] - \tilde{c}_1\, F(\tilde{c}_1)\, \int_{\tilde{c}_1}^{c_H} \frac{[1-F(y)]^{N-1}}{y^2}\, \mathrm{d}y}, \tag{21}$$

where $\tilde{c}_1$ *is defined as*

$$\tilde{c}_1 \equiv \inf\left\{c \in [c_L, c_H] : h(c) + \frac{c^2\, w(c)\, f(c)^2}{F(c) + c\, f(c)} = h(c_H)\right\},$$

the optimal reduced-form mechanism is given as follows:

$$Q(c) = \begin{cases} 1, & \forall c \in [c_L, \bar{c}], \\ B\left[\frac{[1-F(c)]^{N-1}}{c} - \int_c^{\check{c}} \frac{[1-F(y)]^{N-1}}{y^2}\, \mathrm{d}y\right], & \forall c \in (\bar{c}, \check{c}], \\ 0, & \forall c \in (\check{c}, c_H], \end{cases} \tag{22}$$

and

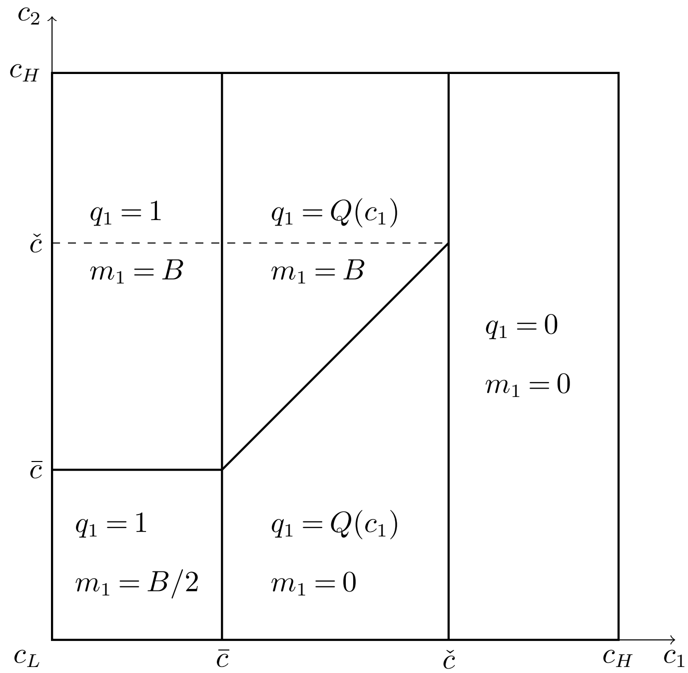
$$M(c) = \begin{cases} \bar{c}\left[B \int_{\bar{c}}^{\check{c}} \frac{[1-F(y)]^{N-1}}{y^2}\, \mathrm{d}y + 1\right], & \forall c \in [c_L, \bar{c}], \\ B\,[1 - F(c)]^{N-1}, & \forall c \in (\bar{c}, \check{c}], \\ 0, & \forall c \in (\check{c}, c_H], \end{cases} \tag{23}$$

where the unique threshold pair $(\bar{c}, \check{c})$ *satisfying* $c_L < \bar{c} < \check{c} \leq c_H$ *is determined by the system*

$$\begin{cases} [F(\bar{c}) + \bar{c}\, f(\bar{c})]\, [h(\check{c}) - h(\bar{c})] = \bar{c}^2\, w(\bar{c})\, f(\bar{c})^2, \\ \bar{c}\left[B \int_{\bar{c}}^{\check{c}} \frac{[1-F(y)]^{N-1}}{y^2}\, \mathrm{d}y + 1\right] F(\bar{c}) = \frac{B}{N}\left[1 - [1 - F(\bar{c})]^N\right]. \end{cases} \tag{24}$$

When the funder's budget falls within the range specified in Proposition 2, we propose the implementation rule described in Corollary 2. The mechanism is governed by two thresholds, $\bar{c}$ and $\check{c}$, determined by the system of equations in (24). If firm i reports a cost below $\bar{c}$, it shares the total budget B equally with all firms submitting similarly low-cost reports, and the funder claims the

Figure 2 **Illustration of the implementation rule defined in Corollary 2 for the case $N=2$. We use $q_1 \equiv q(c_1,c_2)$ and $m_1 \equiv m(c_1,c_2)$ to denote the allocation and payment associated with firm 1. Since the mechanism is symmetric, the implementation for firm 2 is omitted for simplicity.**



entire project from it. If the reported cost lies between $\bar{c}$ and $\check{c}$, the funder claims a share strictly less than 1, and firm i receives the entire budget B only if no competitor reports a lower cost. In this intermediate range, the firm's expected payoff remains nonnegative, although its ex post payoff will be negative if it fails to secure funding. Finally, if the reported cost exceeds $\check{c}$, the firm receives no funding, and the funder claims nothing. Figure 2 depicts the ex post allocation and compensation scheme when there are two agents; the x- and y-axes correspond to the reported costs of firms 1 and 2, respectively.

Corollary 2. *The reduced-form mechanism defined by* (22) *and* (23) *can be implemented as follows: for any* $(c_i, c_{-i}) \in [c_L, c_H]^N$,

$$q(c_i,c_{-i}) = Q(c_i), \quad m(c_i,c_{-i}) = \begin{cases} B/|S|, & \text{if } c_i \le \bar{c}, \\ B, & \text{if } c_i < \min_{j\in[N]\setminus\{i\}} c_j \text{ and } \bar{c} < c_i \le \check{c}, \\ 0, & \text{otherwise}, \end{cases} \tag{25}$$

where $S \equiv \{k \in [N] : c_k \le \bar{c}\}$, *and* $|\cdot|$ *denotes the cardinality of a set.*

As the funder's budget continues to increase, it claims a strictly positive share from all firms in expectation, that is, $Q(c) > 0$ for all $c \in [c_L, c_H]$. The corresponding optimal mechanism is presented in the following proposition.

Proposition 3. *Recall that* $\tilde{c}$, $\underline{B}$, $\bar{B}$, *and* $\tilde{B}$ *are defined in* (16), (17), (20), *and* (21), *respectively. When one of the following conditions holds:*

1. $\tilde{c} < c_0 = c_H$ *and* $\tilde{B} < B \leq \bar{B} = N\, c_H$*; or*
2. $\tilde{c} = c_H$ *and* $\underline{B} < B \leq \bar{B} = N\, c_H$*,*

the optimal reduced-form mechanism is given as follows:

$$Q(c) = \begin{cases} 1, & \forall c \in [c_L, \bar{c}], \\ B\left[\frac{[1-F(c)]^{N-1}}{c} - \int_c^{c_H} \frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y\right], & \forall c \in (\bar{c}, c_H], \end{cases} \tag{26}$$

and

$$M(c) = \begin{cases} \bar{c}\left[B \int_{\bar{c}}^{c_H} \frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y + 1\right], & \forall c \in [c_L, \bar{c}], \\ B\,[1-F(c)]^{N-1}, & \forall c \in (\bar{c}, c_H], \end{cases} \tag{27}$$

where the threshold $\bar{c}$ *is uniquely determined by*

$$\bar{c}\left[B \int_{\bar{c}}^{c_H} \frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y + 1\right] F(\bar{c}) = \frac{B}{N}\left[1 - [1-F(\bar{c})]^N\right]. \tag{28}$$

The implementation of the reduced-form mechanism in this case closely follows that of the previous setting (see Figure A1(b) in Appendix EC.3 for the case $N = 2$). In this setting, we have $\check{c} = c_H$. If a firm reports a cost below $\bar{c}$, it shares the total budget B equally with all competitors who also report below $\bar{c}$, and the funder claims the entire project. Otherwise, the funder claims a share strictly less than one, and a monetary transfer occurs only if the firm's report is the lowest among all. The formal implementation rule is summarized in Corollary 3.

Corollary 3. *The reduced-form mechanism defined by* (22) *and* (23) *can be implemented as follows: for any* $(c_i, c_{-i}) \in [c_L, c_H]^N$*,*

$$q(c_i, c_{-i}) = Q(c_i), \quad m(c_i, c_{-i}) = \begin{cases} B/|S|, & \text{if } c_i \leq \bar{c}, \\ B, & \text{if } c_i < \min_{j \in [N]\setminus\{i\}} c_j \text{ and } c_i > \bar{c}, \end{cases} \tag{29}$$

where $S \equiv \{k \in [N] : c_k \leq \bar{c}\}$*, and* $|\cdot|$ *denotes the cardinality of a set.*

Finally, when the funder's budget exceeds the level specified in Proposition 3, it is never fully exhausted ex post. In this case, the budget constraint is not binding, and the funder claims the entire project from firms whose reported costs fall below a certain threshold. Payments are designed to ensure that the threshold type is indifferent between participating and opting out. The corresponding optimality conditions are presented in the following proposition.

Proposition 4. *When* $B \geq \bar{B} = \frac{N\, c_0\, F(c_0)}{1-[1-F(c_0)]^N}$*, the optimal reduced-form mechanism is given as follows:*

$$Q(c) = \begin{cases} 1, & \forall c \in [c_L, c_0], \\ 0, & \forall c \in (c_0, c_H], \end{cases} \qquad M(c) = \begin{cases} c_0, & \forall c \in [c_L, c_0], \\ 0, & \forall c \in (c_0, c_H], \end{cases} \tag{30}$$

where the threshold c_0 *is defined by* (19)*.*

The implementation rule in this setting is straightforward: if firm i reports a cost below the threshold c_0, the funder claims the entire project, and the firm shares a budget of $\bar{B}$ equally with all competitors who also report costs below c_0. A graphical illustration for the case $N=2$ is given by Figure A1(c) in Appendix EC.3.

COROLLARY 4. *The reduced-form mechanism defined by* (30) *can be implemented as follows: for any* $(c_i, c_{-i}) \in [c_L, c_H]^N$,

$$q(c_i, c_{-i}) = Q(c_i), \quad m(c_i, c_{-i}) = \begin{cases} \bar{B}/|S|, & \text{if } c_i \leq c_0, \\ 0, & \text{otherwise}, \end{cases} \tag{31}$$

where $S \equiv \{k \in [N] : c_k \leq c_0\}$, *and* $|\cdot|$ *denotes the cardinality of a set.*

In the remainder of the paper, we refer to the mechanisms defined in Propositions 1–4 as "the optimal mechanism."

5. Hypotheses and Experiments

Our theoretical analysis provides the optimal solution for the funder's allocation problem in pharmaceutical R&D. However, agents (firms) do not always make rational decisions in practice, leading to behavioral biases in decision-making. To evaluate the real-world applicability of our theoretical solution, we propose two hypotheses and test them using an incentivized lab experiment in which MTurk workers simulate firms facing different allocation mechanisms and budget conditions, mirroring the setting in Section 3.

5.1. Hypothesis Development

Although the optimal mechanisms proposed in Propositions 1 to 4 theoretically incentivize agents to truthfully report their private costs, real-world behavior may deviate from this optimal outcome due to several cognitive biases.

First, agents are not fully rational because of cognitive limitations, time constraints, and imperfect information. As a result, when mechanisms are overly complex (e.g., involving infinitely many menus), agents may be unable to process all the information required to make optimal decisions, even when the mechanism is designed to induce truth-telling. This behavior is consistent with the theory of bounded rationality (Simon 1955, 1979, Camerer et al. 2004b). Second, agents may form incorrect beliefs about how a mechanism works and then make strategic decisions based on those misbeliefs. Even when mechanisms are designed to incentivize truthful behavior, such misbeliefs can lead to systematic deviations from optimal strategies. Under these circumstances, agents may mistakenly believe that inflating their reports is beneficial. This explanation is supported by the literature on strategic misbeliefs (Kagel and Roth 2000, Costa-Gomes et al. 2001, Camerer et al. 2004a). Third, agents may distrust the reporting behavior of their competitors. They may believe that if others systematically misreport their costs, then reporting truthfully would place them at a disadvantage, thereby discouraging honest reporting. Such behavioral patterns are supported by the literature on misbeliefs about others' strategic behavior (Akerlof 1970, Rees-Jones and Skowronek 2018, McCannon and Minuci 2020). These behavioral biases can undermine the effectiveness of our proposed theoretical mechanism in practice.

To evaluate our proposed mechanism, we compare it against two benchmarks: a threshold mechanism and an FPA-based mechanism. The threshold mechanism funds all projects with reported costs below a fixed threshold, providing each with an equal investment amount. Under the FPA-based mechanism, agents report their costs, and the lowest-cost project receives funding from the government first, with the remaining budget allocated to the project with the second-lowest cost, and so on.

Given our proposed and benchmark mechanisms, we expect the degree of deviation from theoretical outcomes to differ across them. Notable studies are showing that experimental outcomes often align closely with theoretical principles for incentive-compatible mechanisms. For instance, research on storable votes in laboratory settings finds that, although participants may rely on heuristics rather than theoretically optimal strategies, the resulting payoffs and allocation outcomes are very similar to those derived from the theoretical model (Casella et al. 2006). Similarly, experimental work on VCG-type public goods mechanisms has shown that when mechanisms are transparent and easy to understand, participants tend to behave in ways that are consistent with the mechanism's intended design (Healy 2006, Chen and Ledyard 2010). Although agents may not always fully understand theoretical mechanisms within a limited time frame, we attempted to mitigate this by providing detailed instructions and examples of our optimal mechanism to facilitate comprehension. Given our efforts to reduce the cognitive burden, along with the supporting findings, we believe that the reporting behavior observed in the lab is unlikely to deviate substantially from the theoretical design.[5]

Under the threshold mechanism, firms that report costs below a predetermined threshold receive an equal share of the budget (see Appendix EC.1.2 for details). Agents aiming to maximize their expected payoff will report a cost below the threshold when their true cost falls below it. If their true cost exceeds the threshold, they can avoid a loss by reporting a very high value and effectively opting out of the bidding process. As a result, the threshold mechanism may induce substantial discrepancies between reported and true costs. Under the FPA-based mechanism, agents are paid based on their reported costs, so truthful reporting yields a zero expected payoff (see Appendix EC.1.3 for details). Therefore, in equilibrium, agents tend to over-report their costs. Compared

[5] We also include an attention check using a simple question, such as selecting the largest number from a set of four, to ensure that participants make their decisions with logical reasoning.

to the FPA-based mechanism and the threshold mechanism, we expect our optimal mechanism to induce smaller deviations between agents' reported and true costs. Based on the above analysis, we formalize these expectations in Hypothesis 1.

Hypothesis 1 (Reported Costs)

1.1 Under our optimal mechanism, there is no significant difference between agents' reported costs and their true costs.

1.2 Compared to the other two benchmark mechanisms (i.e., the threshold mechanism and the FPA-based mechanism), our optimal mechanism leads to smaller deviations between agents' reported and true costs.

From a mechanism design perspective, our model ensures that truthful reporting is an incentive-compatible strategy for agents. This property allows the principal to allocate resources efficiently and achieve the optimal return. Therefore, as long as behavioral deviations from rationality are limited, our mechanism is theoretically superior to the benchmark mechanisms in maximizing the principal's expected payoff. We formalize this implication in Hypothesis 2.

Hypothesis 2 (Principal's Expected Payoff) *Compared to the other two mechanisms (i.e., the threshold mechanism and the FPA-based mechanism), our optimal mechanism generates higher expected payoffs for the principal.*

5.2. Experimental Design

In our study, we use a 2×3 experimental design to test our main hypotheses. In this lab experiment, subjects act as agents in a simulated pharmaceutical R&D setting. They need to strategically disclose their costs to the principal, mirroring the setup in Section 3.

5.2.1. Participants. A total of 530 participants were recruited via Amazon Mechanical Turk to take part in our online lab experiment. Each participant received a $0.80 participation fee. Participants whose final return exceeded that of both competitors earned an additional $1.20 bonus and were entered into a lottery for a $10 reward. This incentive structure was designed to encourage strategic cost reporting and emulate real-world decision-making scenarios. Participants who failed the attention check were excluded from the remainder of the experiment, and responses with missing data were removed. Ultimately, 396 participants remained in the final sample.

5.2.2. Design and Procedure. At the beginning of the experiment, participants were randomly assigned to one of six groups, each representing a unique combination of allocation mechanism and the principal's budget level. Specifically, we implemented a 2×3 experimental design that varied: (a) the principal's allocation mechanism (our optimal mechanism versus benchmark mechanisms), and (b) the principal's budget level (low, medium, or high). To reflect the setting in which each agent's cost is private information, each participant was randomly assigned a value drawn from a uniform distribution with support $[c_L, c_H]$.

Participants assigned to the benchmark mechanism groups interacted with both the threshold mechanism and the FPA-based mechanism, presented in randomized order. The parameter choices in the experiment were as follows: the number of agents $N = 3$, minimum testing cost $c_L = 100$, maximal testing cost $c_H = 200$, and regulator's expected payoff $R = 220$. We referred to the principal's budget levels as "low," "middle," and "high" for the corresponding values of 60, 200, and 500, respectively.

Next, the experimental interface guided subjects through the background and purpose of our study. First, participants were informed that they would represent an agent company engaged in the funding allocation problem of pharmaceutical R&D. During the financing process, they would interact with a government funder (the principal) and strategically disclose their private R&D costs to maximize profits. This was because the principal allocated funding based on the reported costs of each agent and those of two competitors. Second, we explained that the purpose of the study was to understand how agents disclose their private cost information in R&D when interacting with a governmental principal. These insights would contribute to the field of operations management (OM). Third, we described the payment and incentives. Participants were told: "You will receive \$0.80 for participating in this study. Your goal is to maximize your final return by strategically disclosing your private cost. If your return exceeds that of both competitors, you will receive an additional \$1.20 bonus and be entered into a lottery for a \$10 prize. Please note that only participants who pass the attention check on the next page may proceed and receive the participation fee."

Fourth, participants were introduced to their private cost information. Each agent's R&D cost was drawn from a uniform distribution between 100 and 200 million dollars, which was simplified to a range of 100 to 200 in the experiment. Neither the two competitors nor the principal knows the participant's true cost. The principal may choose to partially fund the agent, with the remaining resources covered by outside sources at the agent's marginal cost, as described in Section 3.

Finally, participants proceeded to the main experiment, having been randomly assigned to one of six mechanism–budget conditions: optimal or benchmark mechanism crossed with low, medium, or high budget levels. Each subject also received a randomly assigned private cost. Participants were informed of the allocation mechanism and the budget condition under which they were operating. They were then presented with theoretical information about the optimal solution for that condition, including the principal's allocation rule and a diagram of the funding allocation curve (Q-curve) under the designed mechanism. Additional details about the experimental instructions and procedure are provided in Appendix EC.2. Participants were reminded that they could report a higher value to pursue greater risk and potential reward, or a lower value to play it safe. Finally, they were asked to input the private cost value they wished to report.

6. Experimental Results

In this section, we analyze the experimental results and test two hypotheses by examining the (absolute) deviations between reported and true private costs, as well as the principal's expected payoff, in Sections 6.1 and 6.2. In addition, we discuss behavioral bias through empirical analysis in Section 6.3.

6.1. Deviations of Reported Cost from True Private Cost

As discussed in Hypothesis 1.1, we expect no significant deviations between agents' reported costs and their true costs under our optimal mechanism. To test this, we conduct paired t-tests to assess whether agents' reported costs differ significantly from their true costs.

Table 1 Statistical Comparison Between Reported Cost and True Private Cost

	Reported Cost ($M_{\text{reported-cost}}$)	True Cost ($M_{\text{true-cost}}$)	Difference	Number of Agents (N)
Low Budget ($B = 60$)	152.258	151.970	0.288 (2.384)	132
Middle Budget ($B = 200$)	148.818	149.197	-0.379 (3.890)	132
High Budget ($B = 500$)	147.758	147.682	0.076 (3.479)	132

Note: (1) *p<0.1; **p<0.05; ***p<0.01; (2) The corresponding p-values for the paired t-tests are 0.33, 0.43, and 0.86 for the low-, middle-, and high-budget conditions, respectively, indicating that we cannot reject the null hypothesis under any budget condition at the 10% significance level.

Table 1 presents the average reported and true costs under different budget conditions, along with the corresponding paired differences and statistical test results. We find that under the low-budget condition, there is no significant difference between reported and true costs ($M_{\text{reported-cost}} = 152.258$, $M_{\text{true-cost}} = 151.970$; $p = 0.33$). Under the medium-budget condition, there is also no significant difference in cost values ($M_{\text{reported-cost}} = 148.818$, $M_{\text{true-cost}} = 149.197$; $p = 0.43$). Similarly, under the high-budget condition, the difference is not significant ($M_{\text{reported-cost}} = 147.758$, $M_{\text{true-cost}} = 147.682$; $p = 0.86$). Based on the above analysis, we find that under all three budget conditions, the reported costs and true private costs are not significantly different at 10% significance level. Hence, the experimental results support Hypothesis 1.1.

In Hypothesis 1.2, we expect that compared to the other two benchmark mechanisms (i.e., the threshold mechanism and the FPA-based mechanism), our optimal mechanism leads to smaller deviations between agents' reported costs and true costs. To test it, we also conduct paired t-tests to evaluate the difference.

Table 2 presents the regression results using the deviations between reported and true costs as the dependent variable, with a treatment dummy for the optimal mechanism. In Column (1) of Panel A, we find that the optimal mechanism treatment (*Optimal M. Treatment*) leads to significantly lower deviations between reported and true costs compared to the threshold mechanism under the low-budget condition ($\beta = -30.4848,\ p < 0.01$). Columns (2) and (3) yield similar results, showing that the optimal mechanism significantly reduces the deviations under both the medium-budget condition ($\beta = -27.682,\ p < 0.01$) and high-budget condition ($\beta = -24.182,\ p < 0.01$).

Panel B examines whether the optimal mechanism results in smaller deviations compared to the FPA-based mechanism. Across Columns (1) to (3), the estimated coefficients on *Optimal M. Treatment* are consistently negative and statistically significant, indicating that the optimal mechanism significantly reduces the deviations in reported costs relative to the FPA-based benchmark (low-budget: $\beta = -11.212,\ p < 0.01$; medium-budget: $\beta = -9.758,\ p < 0.01$; high-budget: $\beta = -10.970,\ p < 0.01$).

To more intuitively illustrate the deviations of reported costs from true costs across budget levels, we also report the average deviations along with standard error bars. Figure 3 shows that under the optimal mechanism, deviations between reported and true costs are substantially lower than those under the two benchmark mechanisms, thereby supporting Hypothesis 1.2.

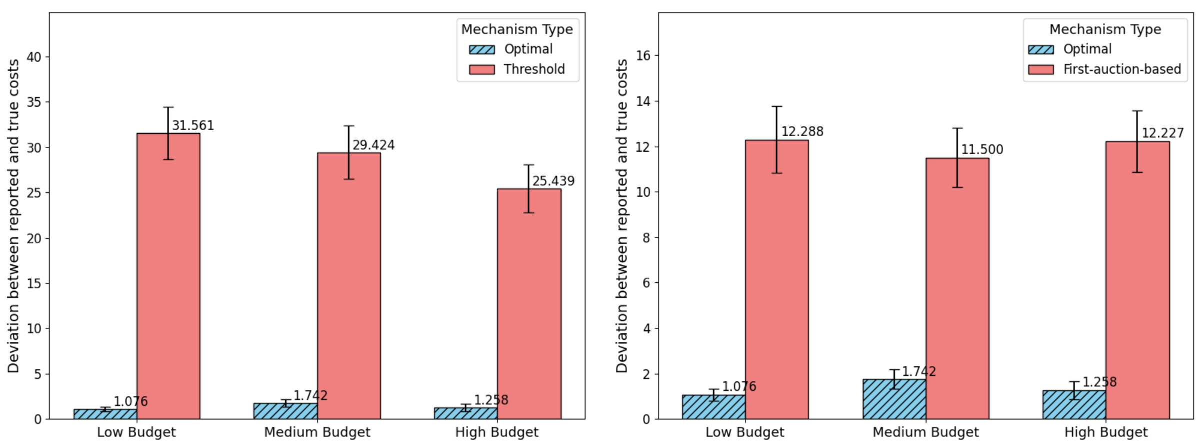
One potential concern is that the observed reduction in deviations between reported and true costs under the optimal mechanism may be driven by changes along the intensive margin, that is, agents over-reporting by a smaller amount when they misreport. In that case, the decrease in deviation would not reflect changes along the extensive margin, namely, fewer agents choosing to over-report their costs (i.e., a higher likelihood of truthful reporting). To address this concern, we re-estimate the regression by replacing the dependent variable with the likelihood of over-reporting costs, while keeping the treatment variable (*Optimal M. Treatment*) unchanged. Table A1 in the appendix presents the estimation results. Across Columns (1) to (3) in Panel A, we find that the coefficients on *Optimal M. Treatment* are all negative and statistically significant. The results show that, compared to the threshold mechanism, the optimal mechanism reduces the likelihood of over-reporting costs by more than 60% across all budget conditions. Similarly, Panel B shows that the optimal mechanism decreases the likelihood of over-reporting by at least 63% compared to the FPA-based mechanism. Figure A2 in the appendix further illustrates the average likelihood of overly reporting costs (with standard error bars) for the three budget levels, comparing our optimal mechanism to the two benchmark mechanisms. Therefore, in light of agents' tendency to over-report costs, the empirical evidence continues to support Hypothesis 1.2.

Table 2 **Deviations between Reported and True Costs – Optimal vs. Benchmark Mechanism Treatments**

	(1)	(2)	(3)
	Deviations between Reported and True Costs		
	Low Budget	Medium Budget	High Budget
Panel A: Optimal vs. Threshold Mechanism Treatments			
Optimal M. Treatment	-30.485***	-27.682***	-24.182***
	(2.892)	(2.986)	(2.661)
Constant	31.561***	29.424***	25.439***
	(2.880)	(2.955)	(2.631)
Observations	132	132	132
No. of Subjects	132	132	132
Panel B: Optimal vs. FPA-Based Mechanism Treatments			
Optimal M. Treatment	-11.212***	-9.758***	-10.970***
	(1.497)	(1.377)	(1.409)
Constant	12.288***	11.500***	12.227***
	(1.474)	(1.308)	(1.351)
Observations	132	132	132
No. of Subjects	132	132	132

Note: *** p<0.01, ** p<0.05, * p<0.1. The dependent variable is the deviation between reported and true costs.

Figure 3 **Deviations from True Cost: Optimal vs.Threshold and FPA-Based Mechanism Treatments**



(a) Optimal vs. Threshold Mechanism (b) Optimal vs. FPA-Based Mechanism

6.2. Principal's Expected Payoff

As discussed in Section 6.1, our proposed mechanism is consistent with the theoretical prediction that it elicits truthful information empirically. We then examine its payoff performance relative to the benchmark mechanisms.

To test Hypothesis 2, we first calculate the principal's expected payoff from each agent based on Equation (2).[6] Table 3 reports the estimation results using the principal's expected payoff as

[6] The principal's expected payoff from each agent is U/N, where U is defined by Equation (2).

the dependent variable and the *Optimal M. Treatment* as the independent variable. In Column (2) of Panel A, we observe that the principal's expected payoff is significantly higher under the optimal mechanism compared to the threshold mechanism ($\beta = 18.884,\ p < 0.05$), suggesting that under the medium-budget condition, our optimal mechanism yields a higher expected payoff for the principal compared to the threshold mechanism. However, in Columns (1) and (3) of Panel A, although the estimated coefficients on *Optimal M. Treatment* are positive, they are not statistically significant, indicating that the optimal mechanism does not significantly increase the principal's expected payoff compared to the threshold mechanism under the low- and high-budget conditions.

Table 3 Principal's Expected Payoff – Optimal vs. Benchmark Mechanism Treatments

	(1)	(2)	(3)
	Principal's Expected Payoff		
	Low Budget	Medium Budget	High Budget
Panel A: Optimal vs. Threshold Mechanism Treatments			
Optimal M. Treatment	3.185	18.884**	0.605
	(2.536)	(8.024)	(7.998)
Constant	5.157***	16.948***	36.829***
	(0.347)	(1.249)	(2.086)
Observations	132	132	132
No. of Subjects	132	132	132
Panel B: Optimal vs. FPA-Based Mechanism Treatments			
Optimal M. Treatment	-5.672	-1.368	-17.514**
	(3.617)	(9.304)	(8.401)
Constant	14.014***	37.200***	54.947***
	(2.601)	(4.872)	(3.312)
Observations	132	132	132
No. of Subjects	132	132	132

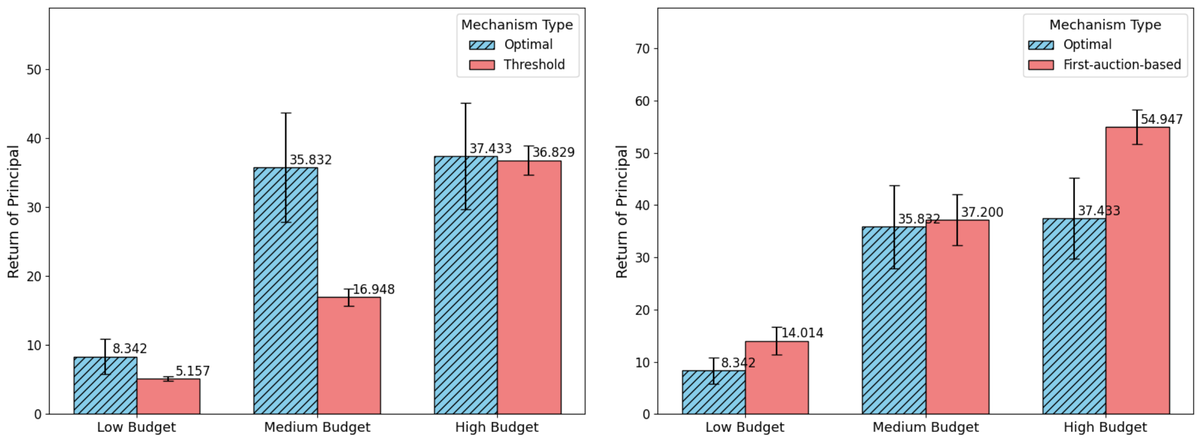
Note: *** p<0.01, ** p<0.05, * p<0.1. The dependent variable is the principal's expected payoff. Each column represents a budget condition.

In Panel B of Table 3, we compare the principal's expected payoff between the optimal and the FPA-based mechanism treatments. In Columns (1) and (2), the estimated coefficients on *Optimal M. Treatment* are not statistically significant, suggesting no significant difference between the two mechanisms. Column (3) shows that the optimal mechanism treatment leads to a lower principal's expected payoff than the FPA-based mechanism under the high-budget condition ($\beta = -17.514,\ p < 0.05$). Figure 4 displays the principal's average expected payoff under the optimal, threshold, and FPA-based mechanisms. These results show that, while the optimal mechanism generally yields higher returns than the threshold mechanism, it does not outperform the FPA-based mechanism in terms of payoff, thereby failing to support Hypothesis 2.

Because the three agents in our experimental setting compete within the same group, we also compute the principal's expected payoff at the group level. Figure A3 in the appendix displays the average group-level principal's expected payoff under the optimal, threshold, and FPA-based mechanisms. The results are consistent with the agent-level analysis above.

Taken together, our findings imply that, although the FPA-based mechanism can also perform well empirically in payoff terms, it induces distorted cost reports and is therefore not well suited to the public sector's objective of eliciting truthful information. By contrast, the optimal mechanism not only elicits truthful cost information but also achieves higher payoffs than the threshold mechanism and payoff levels comparable to those of the FPA-based mechanism, making it better aligned with the public sector's objective of truthful information elicitation while guarantee the effectiveness of funding allocation.

Figure 4 Principal's Expected Payoff: Optimal vs. Two Benchmark Mechanism Treatments



(a) Optimal vs. Threshold Mechanism (b) Optimal vs. FPA-Based Mechanism

6.3. Discussion of Behavioral Bias

As shown by the experimental results in Table 3, the theoretically optimal mechanism does not outperform the FPA-based mechanism in terms of payoff, contrary to Hypothesis 2. However, Section 6.1 shows that deviations between reported and true costs are not statistically significant under the optimal mechanism, indicating no obvious reporting bias. These findings suggest that the main discrepancy arises not because the optimal mechanism fails to elicit truthful information, but because behavioral biases under the FPA-based mechanism allow it to achieve high payoffs while distorting reported cost information. We therefore examine potential behavioral explanations for this pattern, thereby shedding light on the practical usefulness of the FPA-based mechanism for public-sector funders.

In our experimental setting, *risk aversion* and *ambiguity aversion* are two leading explanations for observed deviations from the risk-neutral bidding benchmark (Pratt 1978, Cox et al. 1988, Salo and Weber 1995). Risk aversion (in our first-price auction environment) means that, holding the underlying environment fixed, bidders place extra weight on securing a safer, more certain payoff and are therefore willing to sacrifice some profit margin in order to increase their chance of winning (Pratt 1978). Ambiguity aversion is the tendency to prefer situations in which the environment and opponents' behavior are clearly and transparently specified over situations in which the relevant features of the environment, or the strategies and valuations of other participants, are uncertain or ill-defined (Ellsberg 1961).

The key distinction is whether the underlying environment is viewed as fixed and well specified (risk) or itself uncertain and ill defined (ambiguity). In auction environments, these forces show up when (i) bidders, facing a given environment, tilt toward more certain payoffs by trading profit margin for a higher chance of winning (risk aversion) and (ii) bidders are unsure about rivals' valuations, strategies, or participation, so the relevant outcome chances are themselves uncertain (ambiguity). Both can induce precautionary bidding relative to the risk-neutral benchmark: in standard models of first-price auctions, risk aversion shifts bids toward safer outcomes (Maskin and Riley 1984, Cox et al. 1988), while under ambiguity, bidders tend to choose more cautious bids, producing systematic departures from the benchmark (Salo and Weber 1995). In our study, the ambiguity-aversion explanation suggests that agents in the FPA-based mechanism, facing uncertainty about the reported costs of their competitors, may adopt conservative reporting strategies. The risk-aversion explanation implies that, even when the underlying environment is held fixed, agents may be willing to sacrifice some expected profit in order to increase their chance of receiving funding, adjusting their reports to secure a safer ranking position.

To identify the two underlying behavioral biases that lead participants to underreport their private costs in our FPA-based mechanism, we consider two experimental approaches: priming and structural manipulations. Priming keeps the economic environment fixed while using short, non-deceptive prompts (for example, a brief text or illustrative example) to make particular considerations, such as payoff risk or uncertainty about others' behavior, more salient. These prompts do not alter payoffs, rules, or opponents; they merely shift participants' attention when choosing bids. Structural manipulations, in contrast, modify the environment itself, for example, by replacing human opponents with robots that follow a fixed strategy, or by switching from stochastic to deterministic payments to eliminate risk.

In our context, full structural control is neither feasible nor fully desirable. Ensuring identical environments across groups would require fixing competitors' strategies and fully specifying the game in each condition, thereby removing the human–human strategic interaction central to our research question. Even then, subjects might still perceive probabilities differently across treatments. Priming avoids these issues. All participants face the same mechanism, information structure, and pool of human opponents; only the salience of payoff risk or strategic uncertainty varies through brief, non-deceptive prompts. This approach offers several practical and methodological advantages. It preserves task comparability because everyone faces the same environment and payoff structure, so differences in bids are more naturally interpreted as differences in attitudes or attention rather than as responses to a different game. It keeps the design simpler and less demanding in terms of sample size, since we do not need additional treatment cells with different payment rules or information regimes. Finally, it better reflects realistic policy interventions, where reframing information is typically more practical than redesigning the allocation mechanism itself.

Therefore, we conducted a second laboratory experiment using priming to identify the two behavioral biases that lead to underreporting under the FPA-based mechanism. In this lab experiment, we recruited 310 participants on MTurk to simulate agents deciding how to report their private costs under the FPA-based mechanism.[7] The experiment was conducted under a high-budget condition, in which the optimal mechanism performs statistically worse than the FPA-based mechanism.

We adopted a 2×2 experimental design that exogenously manipulated the salience of risk aversion (high vs. low) and ambiguity aversion (high vs. low), creating four experimental groups representing all combinations of the two factors. The logic of the experiment is as follows: if, after exogenously increasing the salience of risk aversion or ambiguity aversion, participants in the treatment groups report significantly lower private costs than those in the corresponding control groups (where salience was not enhanced), this would indicate that risk aversion or ambiguity aversion is an underlying mechanism that leads participants to report their private costs more conservatively. The experimental procedure and rules were similar to those described in Section 5.2.2. Participants were introduced to the experimental background, rules, and incentive structure, which were the same as in the first experiment. They then read the group-specific information (which primed the relevant attitude), received their randomly assigned private cost, and made their reporting decisions.

Specifically, the risk-aversion nudge used to enhance salience stated: “Choosing to report a conservative (lower) cost may increase your chances of receiving funding, but could reduce your eventual profit if funded. Conversely, reporting a higher cost may lower your chance of being funded but increases the likelihood of achieving a higher profit if you are funded. Please consider these trade-offs when deciding what to report.” The ambiguity-aversion nudge stated: “Your two competitors’ private costs are randomly distributed between 100 and 200, and their reporting strategies are unknown to you. You should consider that the exact reports and strategies of your

[7] A total of 310 participants were randomly assigned to four groups. After applying the attention check and excluding individuals who did not complete the experiment, we obtained 266 valid participants. The final group sizes were 69, 64, 67, and 66 participants, respectively.

competitors are unpredictable when choosing what to report." Specifically, under otherwise identical experimental conditions, the first group received a high-risk-aversion information nudge; the second group received a high-ambiguity-aversion information nudge; the third group received both nudges; and the fourth group served as the control group without any nudge. The first information nudge emphasizes that reporting a higher private cost may reduce the likelihood of receiving funding, thereby inducing a high-risk-aversion treatment. The second information nudge highlights the high uncertainty surrounding competitors' private costs and the unpredictability of their reporting strategies, thereby inducing a high-ambiguity-aversion treatment.

After running the lab experiment on MTurk, we built a regression model to examine the above mechanisms empirically.

$$\text{ReportedCost}_i = \alpha + \beta_R \cdot \text{HighRisk}_i + \beta_A \cdot \text{HighAmb}_i + \beta_{RA} \cdot \text{Both}_i + \gamma \cdot \text{Control}_i + \varepsilon_i, \qquad (32)$$

where the dependent variable ReportedCost_i represents the strategically reported cost by agent i, and the independent dummy variables HighRisk_i, HighAmb_i, and Both_i equal to 1 if the agent received nudges emphasizing high risk aversion only, high ambiguity aversion only, and both, respectively. Since an agent's true private cost directly influences the strategically reported cost, we include it as a control variable in the regression model.

The regression results are presented in Table 4. We find that all estimated coefficients on HighRisk_i, HighAmb_i, and Both_i are negative and statistically significant at the 1% level. This result confirms that both risk aversion and ambiguity aversion lead agents to report their private costs more conservatively under the FPA-based mechanism in our experimental setting.

Specifically, the estimated coefficient on HighRisk_i is -7.631, suggesting that exposure to the high-risk-aversion nudge leads agents to underreport their costs by an average of 7.631 points relative to the benchmark group. The estimated coefficient on HighAmb_i is -7.852, indicating that the high-ambiguity-aversion nudge results in a slightly greater degree of underreporting. The combined informational nudges produce the largest effect, with an estimated coefficient of –8.830, implying the strongest conservative reporting behavior. Finally, consistent with our expectation, the estimated coefficient on the control variable (true private cost) is 0.919 and statistically significant. Taken together, this supplementary laboratory experiment provides empirical evidence that agents facing the FPA-based mechanism in our setting exhibit behavioral biases, specifically risk aversion and ambiguity aversion, that lead them to report their private costs irrationally and conservatively.

These behavioral tendencies help explain why the FPA-based mechanism can perform well in payoff terms while still distorting reported cost information. The FPA-based mechanism should therefore be used with caution by public-sector funders, because it is not well suited to the objective of truthful information elicitation. By contrast, our proposed mechanism performs consistently in both theory and practice and successfully elicits truthful reporting while maintaining strong payoff performance, demonstrating its practical value for public-sector funders.

Table 4 **Examining Risk Aversion and Ambiguity Aversion under FPA-Based Mechanism**

	(1)
	Reported Cost
HighRisk	-7.631***
	(1.376)
HighAmb	-7.852***
	(1.403)
Both	-8.830***
	(1.393)
Control	0.919***
	(0.0173)
Constant	30.08***
	(2.824)
Observations	266
R-squared	0.919

Note: *** $p<0.01$, ** $p<0.05$, * $p<0.1$. The control variable is the agent's true private cost.

7. Conclusions

This paper addresses the funding-allocation challenge governments face in pharmaceutical R&D and proposes a mechanism-based approach supported by both theoretical and experimental analysis. Our mechanism enables governments to elicit truthful private information from pharmaceutical firms when investing in pharmaceutical R&D, which can in turn support broader public-health objectives by informing future subsidy design and policy interventions. Theoretically, the mechanism induces truthful reporting of private R&D costs while maximizing the government's expected payoff. In practice, however, outcomes may diverge from these theoretical predictions because firms may violate the rationality assumptions underlying the model and exhibit behavioral biases such as risk aversion and ambiguity aversion. Our laboratory evidence shows that the theoretically optimal mechanism performs largely in line with its predictions in practice. In particular, it remains effective in eliciting truthful cost reporting while maintaining strong payoff performance. By contrast, although the first-price-auction-based mechanism can perform similarly in payoff, its empirical performance is driven by systematically distorted reporting behavior. Additional experimental evidence suggests that risk aversion and ambiguity aversion play an important role in generating these distortions.

Based on these findings, our mechanism offers a promising tool for governments, other public-sector organizations, and health-related NGOs to gather reliable market information in pharmaceutical R&D settings, where firms hold significant informational advantages regarding development costs. The elicited information can, in turn, inform subsidy design, supply curve estimation, and longer-term public-health planning related to pharmaceutical R&D. Furthermore, our findings highlight the importance of accounting for behavioral biases when designing investment mechanisms for drug R&D. In particular, governments should consider experimental validation, such as laboratory testing, before implementing theoretically optimal mechanisms in practice. Such efforts are important for assessing whether a mechanism that is optimal in theory remains effective under realistic behavioral conditions. Overall, our paper provides insights into how public-sector funders can design incentive mechanisms that elicit truthful information from firms engaged in pharmaceutical R&D, thereby supporting more effective innovation policy and public-health decision-making.

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Online Appendix

EC.1. Parameter Calculations in the Experimental Setting

We set $N=3$, $c_L=100$, $c_H=200$, $R=220$, and f is the density of a uniform distribution on $[c_L, c_H]$, i.e., $F(c)=(c-c_L)/(c_H-c_L)$ for any $c\in[c_L,c_H]$.

EC.1.1. Optimal Mechanism

Case 1 (low budget: $B=60$). In this case, we have $\tilde{c}\approx 148.32$, and the corresponding allocation rule is given by

$$q(c_1,c_{-1})=Q(c_1)=\begin{cases}0.012\,c-2.4\,\log(c)+10.33, & \text{if } c_1\le\tilde{c},\\ 0, & \text{otherwise,}\end{cases} \tag{EC.1}$$

and

$$m(c_1,c_{-1})=\begin{cases}B, & \text{if } c_1<\min_{j\in\{2,\ldots,N\}}c_j \text{ and } c_1\le\tilde{c},\\ 0, & \text{otherwise.}\end{cases} \tag{EC.2}$$

Let c_1 denote firm 1's reported type. The mechanism is implemented as follows:

- If $c_1\le\bar{c}$, the principal invests in firm 1's project and claims a share $Q(c_1)$ of the project's return. Additionally, if firm 1 reports the lowest type among all participating agents, it receives a payment of B from the principal.
- If $c_1>\bar{c}$, the principal does not claim firm 1's project, and firm 1 receives no compensation.

In this scenario, the principal's optimal expected payoff is given by

$$\frac{3}{100}\int_{100}^{148.32}\left[220\,[0.012\,c-2.4\,\log(c)+10.33]-60\left(\frac{200-c}{100}\right)^2\right]\mathrm{d}c\approx 32.31.$$

Case 2 (medium budget: $B=200$). In this case, we have $\bar{c}\approx 132.7$ and $\check{c}\approx 153$, and the corresponding allocation rule is given by

$$q(c_1,c_{-1})=Q(c_1)=\begin{cases}1, & \forall c_1\in[c_L,\bar{c}],\\ 0.04\,c-8\,\log(c)+34.41, & \forall c_1\in(\bar{c},\check{c}],\\ 0, & \forall c_1\in(\check{c},c_H],\end{cases}$$

and

$$m(c_1,c_{-1})=\begin{cases}B/|S|, & \text{if } c_1\le\bar{c},\\ B, & \text{if } c_1<\min_{j\in\{2,\ldots,N\}}c_j \text{ and } \bar{c}<c_1\le\check{c},\\ 0, & \text{otherwise,}\end{cases}$$

where $S\equiv\{k\in[N]:c_k\le\bar{c}\}$.

Let c_1 denote firm 1's reported type. The mechanism is implemented as follows:

- If $c_1\le\check{c}$, the principal claims a share $Q(c_1)$ of the project;
- If $c_1\le\bar{c}$, firm 1 receives a transfer of B/S from the principal, where S denotes the number of firms whose reported types are below $\bar{c}$;
- If $\bar{c}<c_1\le\check{c}$, firm 1 receives a transfer of B from the principal only if its reported cost is the lowest among all N agents;
- If $c_1>\check{c}$, the principal does not claim firm 1's project, and firm 1 receives no payment.

In this scenario, the principal's optimal expected payoff is given by

$$\frac{3}{100}\left\{\int_{100}^{132.7}\left[220-132.7\left[\int_{132.7}^{153}\frac{1}{y^2}\left(\frac{200-y}{100}\right)^2\mathrm{d}y+1\right]\right]\mathrm{d}c\right.$$
$$\left.+\int_{132.7}^{153}\left[220\,[0.04\,c-8\,\log(c)+34.41]-200\left(\frac{200-c}{100}\right)^2\right]\mathrm{d}c\right\}\approx 103.86.$$

Case 3 (high budget: $B = 500$). In this case, we have $\check{c} = 160$, and the corresponding allocation rule is given by

$$q(c_1, c_{-1}) = Q(c_1) = \begin{cases} 1, \ \forall c_1 \in [c_L, \check{c}], \\ 0, \ \forall c_1 \in (\check{c}, c_H], \end{cases} \qquad m(c_1, c_{-1}) = \begin{cases} 307.7/|S|, & \text{if } c_1 \leq \check{c}, \\ 0, & \text{otherwise}, \end{cases}$$

where $S \equiv \{k \in [N] : c_k \leq \check{c}\}$.

Let c_1 denote firm 1's reported type. The mechanism is implemented as follows:

- If $c_1 \leq \check{c}$, the principal claims the entire project and provides a transfer of $\bar{B}/S$ to firm 1, where $\bar{B}$ is a constant defined in Proposition 4 and S denotes the number of firms whose reported types are below $\check{c}$;
- If $c_1 > \check{c}$, the principal does not claim firm 1's project, and firm 1 receives no compensation.

In this scenario, the principal's optimal expected payoff is given by

$$\frac{3}{100} \int_{100}^{160} (220 - 160)\, \mathrm{d}c = 108.$$

EC.1.2. Threshold Mechanism

In this setting, the principal allocates the resource according to a simple threshold mechanism. We fix the threshold at 175.

The rule is as follows: when firm 1's reported cost exceeds the threshold (175), the principal will not claim its project, and the corresponding monetary transfer is 0. Otherwise, firm 1 will share the budget equally with all its competitors whose reports are below 175. The corresponding allocation and payment rules are given by

$$q(c_1, c_{-1}) = \begin{cases} \min\left\{\frac{B}{175\, S}, 1\right\}, & \text{if } c_1 \leq 175, \\ 0, & \text{if } c_1 > 175, \end{cases} \qquad m(c_1, c_{-1}) = 175\, q(c_1, c_{-1}),$$

where S is the number of firms reporting below 175. Note that the maximum compensation the firm can receive is 175.

For instance, suppose $B = 200$, and three agents report their costs as 110, 130, and 150, respectively. In this case, the funder claims a share of $200/(3 \cdot 175)$ from each of the three firms, and each firm receives a funding of 200/3.

EC.1.3. FPA-Based Mechanism

In this setting, the principal adopts a first-price auction-style rule to allocate the limited resources.

Let c_i^* denote the i-th lowest report from the firm. After each firm makes its report, the principal allocates as follows:

- The firm reporting the lowest cost (denoted by c_1^*) will be claimed a share of $\min\left\{\frac{B}{c_1^*}, 1\right\}$ and receive $c_1^* \cdot \min\left\{\frac{B}{c_1^*}, 1\right\}$.
- If there is additional budget after the first round of allocation, the firm reporting the second lowest cost (denoted by c_2^*) will be claimed a share of $\min\left\{\frac{B - c_1^*}{c_2^*}, 1\right\}$ and receive $c_2^* \cdot \min\left\{\frac{B - c_1^*}{c_2^*}, 1\right\}$.
- The principal repeats this procedure until the budget is exhausted or all the N agents' requests are satisfied.

For instance, suppose $B = 200$, and three agents report their costs as 110, 130, and 150, respectively. In this case, the agent reporting 110 will be claimed the whole project and receive 110, and the agent reporting 130 will be claimed a share $(200 - 110)/130$ of its project and receive the remaining 90. Since the budget is exhausted, the agent reporting 150 will not develop the project for the principal.

EC.2. Detailed Experimental Procedure

EC.2.1. Introduction to the Background of Experiment

We first present the background of the experiment and the purpose of the study to the participants, as follows:

You are representing an agent company engaged in the research and development (R&D). During the funding process, you will interact with a government principal (represented by our research team) and must strategically disclose your R&D costs (i.e., private costs) to maximize your profits. This is because the government principal allocates investment funds based on your disclosed costs as well as those disclosed by two competing agents.

The purpose of this study is to understand how agents strategically disclose private costs in the context of pharmaceutical R&D, under different funding allocation mechanisms introduced by the principal. We expect that the findings will contribute to research in operations management and healthcare. Participation in this study is expected to take approximately 15 minutes.

Payment. First, you will receive 80 cents for participating in this study. Your goal is to maximize your final return through strategically disclosing your cost. If your return exceeds that of both your competitors, you will receive an additional $1.2 reward and be entered into a lottery for a $10 bonus. Please note that only participants who pass the simple attention check on the following page can proceed with the experiment and receive the participation fee.

Cost-related Information. (1) Each simulated agent's R&D cost is uniformly distributed between 100 million and 200 million, which is simplified as 100–200 in the experiment. (2) You will receive your own private R&D cost at the beginning of the study. (3) Neither your two competitors nor the principal (represented by the researchers) will know your true cost during the experiment. (4) Even if the principal's total budget is lower than your true cost, it doesn't matter — the investor only claims part of the project, and you will receive the full amount of funding from other sources, meaning Medicine will still be successfully developed.

Informed Consent. By clicking "yes" below, you are indicating that you consent to participate in this research study.

EC.2.2. Attention Check

We then administer an attention check using a single-choice question, asking participants to select the largest number among four randomly ordered numerical options. This attention check helps us screen out participants who are not reading carefully and are merely attempting to complete the study for compensation.

EC.2.3. Main Instruction

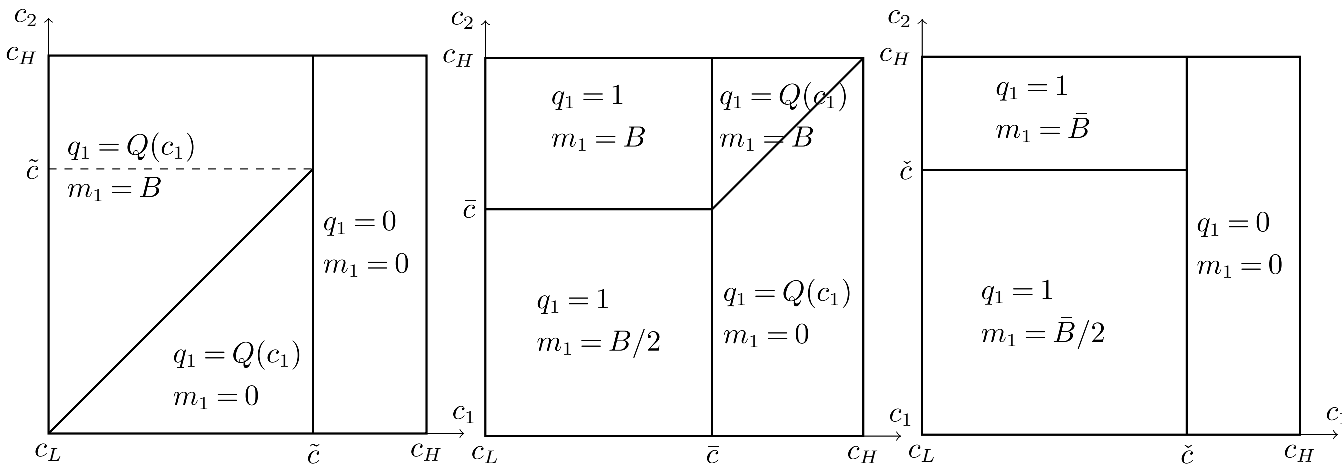
Each subject was randomly assigned to one of six groups: the optimal mechanism–low budget group, optimal mechanism–medium budget group, optimal mechanism–high budget group, benchmark mechanism–low budget group, benchmark mechanism–medium budget group, and benchmark mechanism–high budget group.[8] The low, medium, and high budget levels were set at 50, 200, and 500, respectively. In addition, each subject received a randomly assigned private cost, which was an integer between 100 and 200.

The total value generated for the principal from the completion of a project was fixed at 220. Subjects were then informed about the specific allocation mechanism and budget condition they were operating under. They were also presented with theoretical information regarding the optimal strategy in that condition, including the principal's allocation rule and a visual representation of the funding allocation curve under the corresponding mechanism. Details of the principal's allocation rules under the optimal mechanism, the threshold mechanism, and the FPA-based mechanism are provided in Appendix Section EC.1. The funding allocation curve under our optimal mechanism is illustrated in Figure 1.

[8] When agents were assigned to the benchmark mechanism group, they were presented with the threshold and FPA-based mechanism conditions in a random order, and were asked to input their reported cost under each condition.

For example, under the optimal mechanism, agents can understand that truthfully reporting their private cost would yield the highest theoretical profit. In contrast, under the FPA-based mechanism, they recognize the importance of strategically reporting slightly higher costs to increase their ex post payoffs upon receiving funding, while avoiding excessive over-reporting that could reduce their chances of being funded. Finally, each subject was asked to write down the private cost they chose to report strategically.

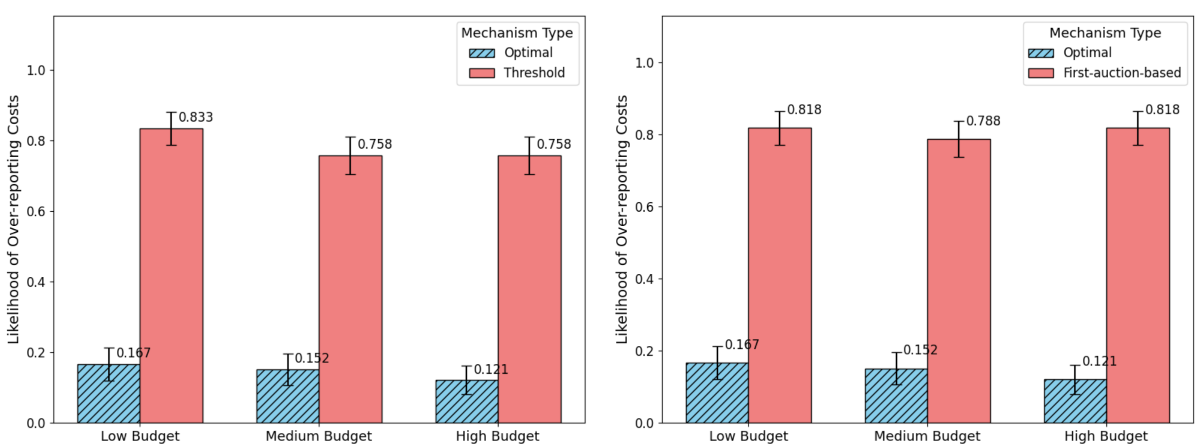
EC.3. Additional Tables and Figures



(a) Low B implementation (b) High B implementation (c) Very high B implementation

Figure A1 Implementation rules for the case $N = 2$ under different budget regimes.

Figure A2 Likelihood of Over-reporting Costs: Optimal vs. Threshold and FPA-Based Mechanism Treatments



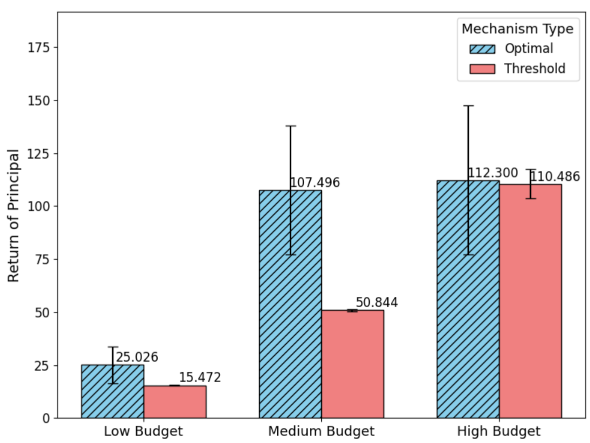
(a) Optimal vs. Threshold Mechanism (b) Optimal vs. FPA-Based Mechanism

Table A1 Likelihood of Over-reporting Costs – Optimal vs. Benchmark Mechanism Treatments

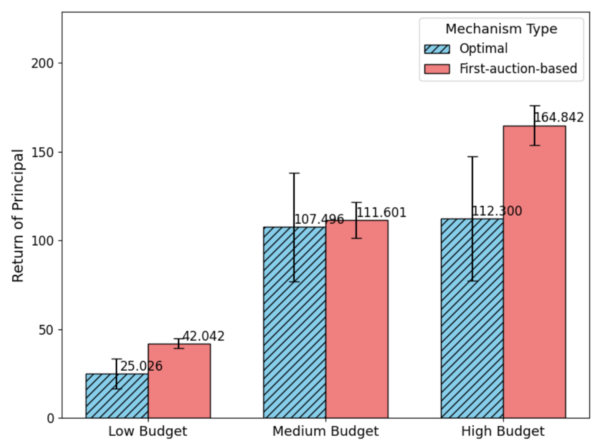
	(1)	(2)	(3)
	Likelihood of Over-reporting Costs		
	Low Budget	Medium Budget	High Budget
Panel A: Optimal vs. Threshold Mechanism Treatments			
Optimal M. Treatment	-0.6667***	-0.6061***	-0.6364***
	(0.065)	(0.069)	(0.067)
Constant	0.8333***	0.7576***	0.7576***
	(0.046)	(0.053)	(0.053)
Observations	132	132	132
No. of Subjects	132	132	132
Panel B: Optimal vs. FPA-Based Mechanism Treatments			
Optimal M. Treatment	-0.6515***	-0.6364***	-0.6970***
	(0.067)	(0.067)	(0.063)
Constant	0.8182***	0.7879***	0.8182***
	(0.048)	(0.051)	(0.048)
Observations	132	132	132
No. of Subjects	132	132	132

Note: *** $p<0.01$, ** $p<0.05$, * $p<0.1$. The dependent variable is the deviation between reported and true costs.

Figure A3 Principal's Expected Payoff (Group Level): Optimal vs. Two Benchmark Mechanism Treatments



(a) Optimal vs. Threshold Mechanism



(b) Optimal vs. FPA-Based Mechanism

EC.4. Proofs

The following lemma is standard in mechanism design settings with direct payments between the principal and the agents. For completeness, we state it here.

LEMMA EC.1. *The incentive constraints* (IC) *and* (IR) *hold if and only if*

$$Q \text{ non-increasing}, \tag{EC.3}$$

$$M(c) = u(c_H) + \int_c^{c_H} Q(y)\,\mathrm{d}y + cQ(c), \text{ and} \tag{EC.4}$$

$$u(c_H) = 0, \tag{EC.5}$$

where $u(c) \equiv \hat{u}(c,c)$.

Proof of Lemma EC.1. This result is standard. See Proposition 5.2 on page 66 of Krishna (2009). □

PROPOSITION EC.1. *The funder's optimization problem* (12) *can be rewritten as follows:*

$$\max_{Q\in\Omega'} \int_{c_L}^{c_H} w(c)\, Q(c)\, f(c)\, \mathrm{d}c, \tag{EC.6}$$

where the virtual valuation function w *is defined by*

$$w(c) \equiv R - c - \frac{F(c)}{f(c)}, \quad \forall c \in [c_L, c_H],$$

and the feasible set Ω' *is determined by linear constraints* (10),(EC.3)*, and*

$$\int_{c_L}^{c} \left[\int_{y}^{c_H} Q(z)\,\mathrm{d}z + y\,Q(y)\right] \mathrm{d}F(y) \leq \frac{B}{N}\left[1 - [1-F(c)]^N\right], \quad \forall c \in [c_L, c_H].$$

Proof of Proposition EC.1. Consider the funder's problem (12). Substituting M in the objective function with (EC.4) yields

$$\begin{aligned}\int_{c_L}^{c_H} [R\,Q(c) - M(c)]\,\mathrm{d}F(c) &= \int_{c_L}^{c_H} \left[R\,Q(c) - \left[\int_{c}^{c_H} Q(y)\,\mathrm{d}y + c\,Q(c)\right]\right] \mathrm{d}F(c) \\ &= \int_{c_L}^{c_H} \underbrace{\left[R - c - \frac{F(c)}{f(c)}\right]}_{=w(c)} Q(c)\, f(c)\,\mathrm{d}c.\end{aligned}$$

The Border's constraint in (9) follows from substituting the expression for M given in (EC.4). Moreover, the nonnegativity of M is implied by the nonnegativity of Q.

This completes the proof. □

Proof of Proposition 1. First, we verify the feasibility of our candidate solution. For any $c \in [c_L, \tilde{c}]$, differentiating Q yields

$$Q'(c) = -\frac{B\,(N-1)\,[1-F(c)]^{N-2}\, f(c)}{c} < 0.$$

Therefore, the desired feasibility condition boils down to

$$Q(c_L) \leq 1 \quad \Leftrightarrow \quad B \leq \left[\frac{1}{c_L} - \int_{c_L}^{\tilde{c}} \frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y\right]^{-1}$$

and

$$Q(\tilde{c}) \geq 0 \quad \Leftrightarrow \quad \frac{[1-F(\tilde{c})]^{N-1}}{\tilde{c}} \geq 0.$$

Both conditions hold automatically. Since the Border's constraint (9) is binding on $[c_L, \tilde{c}]$, the desired feasibility condition also holds.

Next, we prove the optimality of our candidate solution. Define

$$\begin{aligned}\xi(c) &= -J'(c), && \forall c \in [c_L, \tilde{c}], \\ \delta(c) &= -w(c)\, f(c) + w(\tilde{c})\, f(\tilde{c}), && \forall c \in [\tilde{c}, c_H],\end{aligned}$$

where

$$J(c) \equiv \frac{N(c)}{c^2\, f(c)} \tag{EC.7}$$

and

$$N(c) \equiv c\, w(c)\, f(c) - \tilde{c}\, w(\tilde{c})\, f(\tilde{c}) + \int_c^{\tilde{c}} w(y)\, f(y)\, \mathrm{d}y. \tag{EC.8}$$

We have

$$N(c) = -\int_c^{\tilde{c}} y\, \mathrm{d}\left(w(y)\, f(y)\right) \geq 0,$$

where the monotonicity of $w(c)\, f(c)$ is guaranteed by Assumption 1, i.e.,

$$\frac{\mathrm{d}}{\mathrm{d}c}\left[w(c)\, f(c)\right] = (R-c)\, f'(c) - 2\, f(c) < 0.$$

The nonnegativity of ξ is equivalent to the monotonicity of

$$\begin{aligned}\frac{1}{c^2\, f(c)}\left[c\, w(c)\, f(c) - \tilde{c}\, w(\tilde{c})\, f(\tilde{c}) + \int_c^{\tilde{c}} w(y)\, f(y)\, \mathrm{d}y\right] &= \frac{1}{c^2\, f(c)}\left[c\, w(c)\, f(c) - \int_{c_L}^{c} w(y)\, f(y)\, \mathrm{d}y\right] \\ &= \frac{1}{c^2\, f(c)}\left[c\, w(c)\, f(c) - (R-c)\, F(c)\right],\end{aligned}$$

We have

$$\begin{aligned}\frac{1}{c^2\, f(c)}\left[c\, w(c)\, f(c) - (R-c)\, F(c)\right] \text{ non-increasing} \quad &\Leftrightarrow \quad 2\, F(c)\, f(c) + c\, F(c)\, f'(c) - 2\, c\, f(c)^2 \leq 0, \\ &\Leftarrow \quad 2\, f(c) + c\, f'(c) \leq 0.\end{aligned}$$

Moreover, we have

$$\frac{N(c)}{c^2\, f(c)} \text{ non-increasing} \quad \Leftrightarrow \quad c^2\, f(c) \underbrace{N'(c)}_{= c\, \frac{\mathrm{d}}{\mathrm{d}c}(w(c)\, f(c)) \leq 0} - c\, N(c)\, \left[2\, f(c) + c\, f'(c)\right] \leq 0,$$

which also holds when $2\, f(c) + c\, f'(c) \geq 0$. Therefore, we conclude that ξ is nonnegative on $[c_L, \tilde{c}]$.

The nonnegativity of δ follows from the monotonicity of $w(c)\, f(c)$ directly.

For any $c \in [c_L, \tilde{c}]$, we have

$$\xi(c) = -J'(c) \quad \Rightarrow \quad \int_c^{\tilde{c}} \xi(y)\, dy = J(c),$$

where the last step follows from $J(\tilde{c}) = 0$. Therefore, we have

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}c}\left[\int_{c_L}^{c}\left(\int_y^{\tilde{c}} \xi(z)\, \mathrm{d}z\right) f(y)\, \mathrm{d}y + c\, f(c)\int_c^{\tilde{c}} \xi(y)\, \mathrm{d}y\right] \\ =\ &\left[2\, f(c) + c\, f'(c)\right]\int_c^{\tilde{c}} \xi(y)\, \mathrm{d}y - c\, f(c)\, \xi(c) \\ =\ &\left[2\, f(c) + c\, f'(c)\right]\, J(c) - c\, f(c)\, \xi(c) \\ =\ &\frac{-c^2\, w(c)\, f(c)^2 + c^3\, w'(c)\, f(c)^2 + 2\, c\, (R-c)\, F(c)\, f(c) + c^2\, F(c)\, f(c) + c^2 (R-c)\, F(c)\, f'(c) - c^2 (R-c)\, f(c)^2}{c^3\, f(c)} \\ &+ \frac{2\, f(c) + c\, f'(c)}{c^2\, f(c)}\left[c\, w(c)\, f(c) - (R-c)\, F(c)\right] \\ =\ &(R-c)\, f'(c) - 2\, f(c) \\ =\ &\frac{\mathrm{d}}{\mathrm{d}c}\left[w(c)\, f(c)\right],\end{aligned}$$

where the fourth equality follows from substituting the definition of w and simplifying the resulting expression. Since

$$\left[\int_{c_L}^{c}\left(\int_{y}^{\tilde{c}}\xi(z)\,\mathrm{d}z\right)f(y)\,\mathrm{d}y+c\,f(c)\int_{c}^{\tilde{c}}\xi(y)\,\mathrm{d}y\right]_{c=c_L}=c_L\,f(c_L)\,J(c_L)=w(c_L)\,f(c_L),$$

we know that

$$\int_{c_L}^{c}\left(\int_{y}^{\tilde{c}}\xi(z)\,\mathrm{d}z\right)f(y)\,\mathrm{d}y+c\,f(c)\int_{c}^{\tilde{c}}\xi(y)\,\mathrm{d}y=w(c)\,f(c),\quad\forall c\in[c_L,\tilde{c}].$$

For any $c\in[\tilde{c},c_H]$, by definition, we have

$$\int_{c_L}^{\tilde{c}}\left(\int_{y}^{\tilde{c}}\xi(z)\,\mathrm{d}z\right)f(y)\,\mathrm{d}y-\delta(c)=w(c)\,f(c).$$

Because

$$\int_{c_L}^{\tilde{c}}\left[\int_{c_L}^{c}\left[\int_{y}^{c_H}Q(z)\,\mathrm{d}z+y\,Q(y)\right]\mathrm{d}F(y)\right]\xi(c)\,\mathrm{d}c+\int_{\tilde{c}}^{c_H}[-Q(c)]\,\delta(c)\,\mathrm{d}c=\int_{c_L}^{c_H}w(c)\,Q(c)\,f(c),$$

the objective function is bounded above by

$$\frac{B}{N}\int_{c_L}^{\tilde{c}}\left[1-[1-F(c)]^N\right]\xi(c)\,\mathrm{d}c.$$

Our candidate solution achieves this upper bound. Subtracting the candidate objective value from the upper bound, applying integration by parts to $\int_{c_L}^{\tilde{c}}\left[1-[1-F(c)]^N\right]\mathrm{d}J(c)$, and substituting the definitions of Q yields

$$\begin{aligned}&\frac{B}{N}\int_{c_L}^{\tilde{c}}\left[1-[1-F(c)]^N\right]\xi(c)\,\mathrm{d}c-\int_{c_L}^{\tilde{c}}w(c)\,Q(c)\,f(c)\,\mathrm{d}c\\=&-B\int_{c}^{\tilde{c}}\frac{1}{c^2}(R-c)\,F(c)\,[1-F(c)]^{N-1}\,\mathrm{d}c+B\int_{c_L}^{\tilde{c}}(R-y)\,F(y)\,\frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y\\=&\;0,\end{aligned}$$

and thus establishes its optimality.

This completes the proof. □

Proof of Corollary 1. By construction, for any $c_1\in[c_L,c_H]$, we have

$$\int_{c_{-i}\in[c_L,c_H]^{N-1}}q(c_i,c_{-i})\prod_{k\in[N]\setminus\{i\}}\mathrm{d}F(c_k)=Q(c_i)$$

and

$$\int_{c_{-i}\in[c_L,c_H]^{N-1}}m(c_i,c_{-i})\prod_{k\in[N]\setminus\{i\}}\mathrm{d}F(c_k)=B\,[1-F(c_i)]^{N-1}=M(c_i).$$

Meanwhile, for any reporting profile $\mathbf{c}\in[c_L,c_H]^N$, we have $\sum_{i=1}^{N}m(c_i,c_{-i})\le B$ and $0\le q(c_i,c_{-i})\le 1$, implying its feasibility.

This completes the proof. □

Proof of Proposition 2. We begin by verifying the feasibility of our candidate solution, starting with the existence of the thresholds $\bar{c}$ and $\check{c}$. The first equation in (24), which characterizes the relationship between $\bar{c}$ and $\check{c}$, can be rewritten as

$$h(\check{c}) = h(\bar{c}) + \frac{\bar{c}^2\, w(\bar{c})\, f(\bar{c})^2}{F(\bar{c}) + \bar{c}\, f(\bar{c})}.$$

Since h is monotonic, for any fixed $\bar{c}$, there exists a unique $\check{c}$ satisfying this equation. Let $g(\bar{c})$ denote this solution. That is,

$$h(g(\bar{c})) = h(\bar{c}) + \frac{\bar{c}^2\, w(\bar{c})\, f(\bar{c})^2}{F(\bar{c}) + \bar{c}\, f(\bar{c})} \quad \Rightarrow \quad g(\bar{c}) = h^{-1}\left(h(\bar{c}) + \frac{\bar{c}^2\, w(\bar{c})\, f(\bar{c})^2}{F(\bar{c}) + \bar{c}\, f(\bar{c})}\right).$$

We then prove the monotonicity of function g. We have

$$\begin{aligned} h(c) + \frac{c^2\, w(c)\, f(c)^2}{F(c) + c\, f(c)} &= -c\, w(c)\, f(c) + (R - c)\, F(c) + \frac{c^2\, w(c)\, f(c)^2}{F(c) + c\, f(c)} \\ &= R\, \underbrace{\left[F(c) - c\, f(c) + \frac{c^2 f(c)^2}{c\, f(c) + F(c)}\right]}_{\equiv \Upsilon(c)}. \end{aligned}$$

Differentiating Υ yields

$$\Upsilon'(c) = -c\, f'(c) + \frac{[2\, c\, f(c)^2 + 2\, c^2\, f(c)\, f'(c)]\; [c\, f(c) + F(c)] - c^2\, f(c)^2\; [2\, f(c) + c\, f'(c)]}{[c\, f(c) + F(c)]^2} > 0,$$

where the inequality follows from

$$\begin{aligned} &- c\, f'(c)\, [c\, f(c) + F(c)]^2 + \left[2\, c\, f(c)^2 + 2\, c^2\, f(c)\, f'(c)\right]\; [c\, f(c) + F(c)] - c^2\, f(c)^2\; [2\, f(c) + c\, f'(c)] \\ &= c\, F(c)\; \left[-F(c)\, f'(c) + 2\, f(c)^2\right] \\ &> 0. \end{aligned}$$

Combining the monotonicity of the function $h(c) + \frac{c^2\, w(c)\, f(c)^2}{F(c) + c\, f(c)}$ with that of h yields the desired result.

Define

$$B(\bar{c}) = \frac{\bar{c}\, F(\bar{c})}{\frac{1}{N}[1 - [1 - F(\bar{c})]^N] - \bar{c}\, F(\bar{c})\, \int_{\bar{c}}^{g(\bar{c})} \frac{[1 - F(y)]^{N-1}}{y^2}\, \mathrm{d}y},$$

which represents the unique budget level for which the second equation in (24) holds, given the threshold pair $(\bar{c}, g(\bar{c}))$. We examine the range of $B(\bar{c})$ for $\bar{c} \in (c_L, c_0]$. If $c_0 < c_H$, we observe the following:

- When $\bar{c} \to c_L^+$, we have $g(\bar{c}) \to \tilde{c}$, and hence, $B(\bar{c}) \to \left[\frac{1}{c_L} - \int_{c_L}^{\tilde{c}} \frac{[1-F(y)]^{N-1}}{y^2}\, dy\right]^{-1} = \underline{B}$.
- When $\bar{c} \to c_0^-$, since $w(c_0) = 0$, we have $g(\bar{c}) \to c_0$, and hence, $B(\bar{c}) \to \frac{N c_0 F(c_0)}{1 - [1 - F(c_0)]^N} = \bar{B}$.

By continuity, for any budget level $\underline{B} < B < \bar{B}$, there exists a threshold pair $(\bar{c}, \check{c})$ that satisfies the system of equations (24).

When $\tilde{c} < c_0 = c_H$, we observe the following:

- When $\bar{c} \to c_L^+$, we have $g(\bar{c}) \to \tilde{c}$, and hence, $B(\bar{c}) \to \left[\frac{1}{c_L} - \int_{c_L}^{\tilde{c}} \frac{[1-F(y)]^{N-1}}{y^2}\, dy\right]^{-1} = \underline{B}$.
- When $\bar{c} = \tilde{c}_1$, we have $g(\bar{c}) = c_H$, and hence, $B(\bar{c}) \to \tilde{B}$.

By continuity, for any budget level $\underline{B} < B < \tilde{B}$, there exists a threshold pair $(\bar{c}, \check{c})$ that satisfies the system of equations (24). Combining the arguments above establishes the existence of a threshold pair $(\bar{c}, \check{c})$ under the given conditions.

To prove the uniqueness of the threshold pair, it suffices to show that the map $\bar{c} \mapsto B(\bar{c})$ is strictly monotonic on the relevant interval. It is convenient to work with the reciprocal

$$\phi(\bar{c}) \equiv \frac{1}{B(\bar{c})} = \underbrace{\frac{1-[1-F(\bar{c})]^N}{N\,\bar{c}\,F(\bar{c})}}_{\equiv T_1(\bar{c})} - \underbrace{\int_{\bar{c}}^{g(\bar{c})} \frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y}_{\equiv T_2(\bar{c})},$$

and show $\phi'(\bar{c}) < 0$. Taking the derivative yields

$$T_1'(\bar{c}) = \frac{N\,[1-F(\bar{c})]^{N-1} f(\bar{c})\,\bar{c}\,F(\bar{c}) - [1-[1-F(\bar{c})]^N]\;[F(\bar{c}) + \bar{c}\,f(\bar{c})]}{N\,\bar{c}^2\,F(\bar{c})^2},$$

$$T_2'(\bar{c}) = \frac{[1-F(g(\bar{c}))]^{N-1}}{g(\bar{c})^2}\,g'(\bar{c}) - \frac{[1-F(\bar{c})]^{N-1}}{\bar{c}^2}.$$

Therefore, we have

$$\phi'(\bar{c}) = \underbrace{T_1'(\bar{c}) + \frac{[1-F(\bar{c})]^{N-1}}{\bar{c}^2}}_{\equiv A(\bar{c})} - \underbrace{\frac{[1-F(g(\bar{c}))]^{N-1}}{g(\bar{c})^2}\,g'(\bar{c})}_{\equiv D(\bar{c}) \geq 0}.$$

Putting A over a common denominator yields

$$A(\bar{c}) = \frac{[F(\bar{c}) + \bar{c}\,f(\bar{c})]\;[N\,F(\bar{c})\,[1-F(\bar{c})]^{N-1} - [1-[1-F(\bar{c})]^N]]}{N\,\bar{c}^2\,F(\bar{c})^2}.$$

Applying the geometric-series identity $1-(1-F)^N = F\sum_{k=0}^{N-1}(1-F)^k$ yields

$$1-(1-F)^N - N\,F\,(1-F)^{N-1} = F\sum_{k=0}^{N-2}\left[(1-F)^k - (1-F)^{N-1}\right] > 0$$

for any $F \in (0,1)$ and $N \geq 2$, since each summand is nonnegative and the term at $k=0$ is strictly positive. Because $B > \underline{B}$ implies $\bar{c} > c_L$, we have $F(\bar{c}) > 0$. Therefore, $A(\bar{c}) < 0$, which ensures that $\phi'(\bar{c}) = A(\bar{c}) - D(\bar{c}) < 0$ for all $\bar{c} \in (c_L, c_H)$.

Hence, ϕ is strictly decreasing, and $B(\bar{c}) = 1/\phi(\bar{c})$ is strictly increasing. Because $B(\cdot)$ is a continuous and strictly monotonic bijection, for any budget B in the open intervals—namely, $B \in (\underline{B}, \bar{B})$ under condition (i) and $B \in (\underline{B}, \tilde{B})$ under condition (ii)—the equation $B(\bar{c}) = B$ admits a unique solution $\bar{c}$. Consequently, the corresponding $\check{c} = g(\bar{c})$ is also unique, establishing the uniqueness of the threshold pair $(\bar{c}, \check{c})$.

Next, we will verify the feasibility of Q and M, given the existence of $(\bar{c}, \check{c})$. In the proof of Proposition 1, we have established the monotonicity of Q on $[\bar{c}, \check{c}]$. The desired feasibility condition for Q boils down to

$$Q(\bar{c}^+) \leq 1 \quad \Leftrightarrow \quad \frac{[1-F(\bar{c})]^{N-1}}{\bar{c}} - \int_{\bar{c}}^{\check{c}} \frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y \leq \frac{1}{B}$$

and

$$Q(\check{c}) \geq 0 \quad \Leftrightarrow \quad \frac{B\,[1-F(\check{c})]^{N-1}}{\check{c}} \geq 0,$$

where the second inequality holds automatically. To prove the first inequality, because

$$\int_{\bar{c}}^{\check{c}} \frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y = \frac{1}{N\,\bar{c}\,F(\bar{c})}\,\left[1-[1-F(\bar{c})]^N\right] - \frac{1}{B},$$

and hence, the desired feasibility condition can be rewritten as

$$N\,F(\bar{c})\,[1-F(\bar{c})]^{N-1} + [1-F(\bar{c})]^N - 1 \le 0,$$

which has been established when proving $A(\bar{c}) < 0$.

By construction, we have

$$\int_{c_L}^{c} M(y)\,\mathrm{d}y = \frac{B}{N}\,\left[1-[1-F(c)]^N\right]$$

for any $c \in [\bar{c}, \check{c}]$. Moreover, since M is constant on $[c_L, \bar{c}]$, the Border's constraint in (9) continues to hold.

This concludes the proof of primal feasibility. It remains to establish the optimality of our candidate solution. Let

$$\begin{aligned}
&\sigma(c) = w(c)\,f(c) - \frac{[F(c)+c\,f(c)]\,w(\bar{c})\,f(\bar{c})}{F(\bar{c})+\bar{c}\,f(\bar{c})}, && \forall c \in [c_L, \bar{c}],\\
&\xi(c) = -\frac{\mathrm{d}}{\mathrm{d}c}\left[\frac{1}{c^2\,f(c)}\left[c\,w(c)\,f(c) - \check{c}\,w(\check{c})\,f(\check{c}) + \int_c^{\check{c}} w(y)\,f(y)\,\mathrm{d}y\right]\right], && \forall c \in [\bar{c}, \check{c}],\\
&\delta(c) = -w(c)\,f(c) + w(\check{c})\,f(\check{c}), && \forall c \in [\check{c}, c_H].
\end{aligned}$$

With a slight abuse of notation, we continue to use J to denote the function

$$\frac{1}{c^2\,f(c)}\left[c\,w(c)\,f(c) - \check{c}\,w(\check{c})\,f(\check{c}) + \int_c^{\check{c}} w(y)\,f(y)\,\mathrm{d}y\right].$$

Differentiating σ yields

$$\sigma'(c) = \frac{f'(c)\,[F(\bar{c}) + f(\bar{c})\,(\bar{c}-c)] - 2\,f(c)\,f(\bar{c})}{F(\bar{c})+\bar{c}\,f(\bar{c})} < 0,$$

where the inequality is implied by Assumption 1. Therefore, the desired nonnegativity condition reduces to $\sigma(\bar{c}) \ge 0$, which follows directly from its definition. The nonnegativity of ξ is equivalent to the monotonicity of J, which can be established using the same argument as in the proof of Proposition 1. Lastly, the nonnegativity of δ is ensured by the monotonicity of the function $w(c)\,f(c)$.

Because

$$\begin{aligned}
&\sigma(c) + \int_{c_L}^{c}\left(\int_{\bar{c}}^{\check{c}} \xi(z)\,\mathrm{d}z\right) f(y)\,\mathrm{d}y + c\,f(c)\int_{\bar{c}}^{\check{c}} \xi(y)\,\mathrm{d}y = w(c)\,f(c), && \forall c \in [c_L, \bar{c}],\\
&\int_{c_L}^{\bar{c}}\left(\int_{\bar{c}}^{\check{c}} \xi(z)\,\mathrm{d}z\right) f(y)\,\mathrm{d}y + \int_{\bar{c}}^{c}\left(\int_{y}^{\check{c}} \xi(z)\,\mathrm{d}z\right) f(y)\,\mathrm{d}y + c\,f(c)\int_{c}^{\check{c}} \xi(y)\,\mathrm{d}y = w(c)\,f(c), && \forall c \in [\bar{c}, \check{c}], \text{ and}\\
&-\delta(c) + \int_{c_L}^{\bar{c}}\left(\int_{\bar{c}}^{\check{c}} \xi(z)\,\mathrm{d}z\right) f(y)\,\mathrm{d}y + \int_{\bar{c}}^{\check{c}}\left(\int_{y}^{\check{c}} \xi(z)\,\mathrm{d}z\right) f(y)\,\mathrm{d}y = w(c)\,f(c), && \forall c \in [\check{c}, c_H],
\end{aligned}$$

we have

$$\int_{c_L}^{\bar{c}} Q(c)\,\sigma(c)\,\mathrm{d}c + \int_{\bar{c}}^{\check{c}}\left[\int_{c_L}^{c}\left[\int_{y}^{c_H} Q(z)\,\mathrm{d}z + y\,Q(y)\right]\mathrm{d}F(y)\right]\xi(c)\,\mathrm{d}c + \int_{\check{c}}^{c_H}[-Q(c)]\,\delta(c)\,\mathrm{d}c = \int_{c_L}^{c_H} w(c)\,Q(c)\,f(c),$$

and hence, the objective function is bounded above by

$$\int_{c_L}^{\bar{c}} \sigma(c)\,\mathrm{d}c + \frac{B}{N}\int_{\bar{c}}^{\check{c}} \left[1-[1-F(c)]^N\right] \xi(c)\,\mathrm{d}c.$$

Our candidate solution achieves this upper bound. Subtracting the candidate objective value from the upper bound, applying integration by parts to $\int_{\bar{c}}^{\check{c}}\left[1-[1-F(c)]^N\right]\mathrm{d}J(c)$, and substituting the definitions of Q yields

$$B\int_{\bar{c}}^{\check{c}}\left[\frac{\bar{c}\,w(\bar{c})\,F(\bar{c})\,f(\bar{c})}{F(\bar{c})+\bar{c}\,f(\bar{c})} - \check{c}\,w(\check{c})\,f(\check{c}) + \int_{\bar{c}}^{\check{c}} w(y)\,f(y)\,\mathrm{d}y\right]\frac{[1-F(c)]^{N-1}}{c^2}\,\mathrm{d}c = 0,$$

where the equality to zero follows directly from the first equation in (24) that determines the threshold pair $(\bar{c},\check{c})$.

This completes the proof. □

Proof of Corollary 2. By construction, for any $c_1 \in [c_L, c_H]$, we have

$$\int_{c_{-i}\in[c_L,c_H]^{N-1}} q(c_i,c_{-i}) \prod_{k\in[N]\setminus\{i\}} \mathrm{d}F(c_k) = Q(c_i).$$

For any $c\in[c_L,\bar{c}]$, we have

$$\begin{aligned}
\int_{c_{-i}\in[c_L,c_H]^{N-1}} m(c_i,c_{-i}) \prod_{k\in[N]\setminus\{i\}} \mathrm{d}F(c_k) &= \sum_{j=0}^{N-1} \frac{B}{j+1}\binom{N-1}{j} F(\bar{c})^j\,[1-F(\bar{c})]^{N-1-j} \\
&= \sum_{j=0}^{N-1} \frac{B}{j+1}\,\frac{(N-1)!}{j!\,(N-j-1)!} F(\bar{c})^j\,[1-F(\bar{c})]^{N-1-j} \\
&= \frac{B}{N\,F(\bar{c})} \sum_{j=0}^{N-1} \frac{N!}{(j+1)!\,(N-j-1)!} F(\bar{c})^{j+1}\,[1-F(\bar{c})]^{N-1-j} \\
&= \frac{B}{N\,F(\bar{c})} \sum_{k=1}^{N} \binom{N}{k} F(\bar{c})^k\,[1-F(\bar{c})]^{N-k} \\
&= \frac{B}{N\,F(\bar{c})}\left[1-[1-F(\bar{c})]^N\right] \\
&= \bar{c}\left[B\int_{\bar{c}}^{\check{c}} \frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y + 1\right],
\end{aligned}$$

where the last equality follows from the second equation in (24).

For any $c_1 \in [\bar{c},\check{c}]$, we have

$$M(c) = B\cdot \Pr(\text{all its } N-1\text{'s competitors' types are below } c) = B\left[1-[1-F(c)]^{N-1}\right].$$

Meanwhile, for any reporting profile $(c_i,c_{-i})\in[c_L,c_H]^N$, we have $\sum_{i=1}^N m(c_i,c_{-i}) \le B$ and $0 \le q(c_i,c_{-i}) \le 1$, implying its feasibility.

This completes the proof. □

Proof of Proposition 3. We first verify the unique existence of the threshold $\bar{c}$ under the given conditions. Let

$$B(\bar{c}) = \frac{\bar{c}\,F(\bar{c})}{\frac{1}{N}\left[1-[1-F(\bar{c})]^N\right] - \bar{c}\,F(\bar{c})\int_{\bar{c}}^{c_H}\frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y},$$

which represents the budget level at which equation (28) holds for a given threshold $\bar{c}$. To establish uniqueness, we demonstrate that $B(\cdot)$ is strictly increasing. It is convenient to show that its reciprocal, $\phi(\bar{c}) \equiv 1/B(\bar{c})$, is strictly decreasing. Taking the derivative yields

$$\phi'(\bar{c}) = \frac{F(\bar{c}) + \bar{c}\, f(\bar{c})}{N\, \bar{c}^2\, F(\bar{c})^2} \left[N\, F(\bar{c})\, [1 - F(\bar{c})]^{N-1} + [1 - F(\bar{c})]^N - 1 \right].$$

Define $g(x) \equiv N\, x\, (1-x)^{N-1} + (1-x)^N - 1$. Taking the derivative gives $g'(x) = -N\,(N-1)\, x\, (1-x)^{N-2} \le 0$. Since $g(0) = 0$ and $F(\bar{c}) > 0$ for any $\bar{c} > c_L$, we have $g(F(\bar{c})) < 0$. This strictly negative numerator implies $\phi'(\bar{c}) < 0$ for all interior types. Consequently, $B(\cdot)$ is a strictly increasing, continuous function, ensuring that any valid solution $\bar{c}$ to (28) is unique.

For existence, we examine the bounds of the continuous bijection $B(\cdot)$:

• Under condition (1), where $\tilde{c} < c_0 = c_H$, we have $B(\tilde{c}_1) = \tilde{B}$ and $B(c_H) = N\, c_H = \bar{B}$. Therefore, for any budget $B \in (\tilde{B}, \bar{B}]$, there exists a unique $\bar{c} \in (\tilde{c}_1, c_H]$ satisfying (28).

• Under condition (2), where $\tilde{c} = c_H$ and $\underline{B} < B \le \bar{B} = N\, c_H$, we have $\lim_{c \downarrow c_L} B(c) = \underline{B}$ and $B(c_H) = N\, c_H = \bar{B}$. Similarly, for any budget $B \in (\underline{B}, \bar{B}]$, there exists a unique $\bar{c} \in (c_L, c_H]$ satisfying (28).

Next, we establish the feasibility of our candidate solution, given the unique threshold $\bar{c}$. The monotonicity of Q and its feasibility bounds—specifically, $\lim_{c \downarrow \bar{c}} Q(c) \le 1$ and $Q(c_H) \ge 0$—follow directly from the arguments used in the proof of Proposition 2. As for M, since Border's constraint (9) is constructed to bind for all $c \in (\bar{c}, c_H]$, and $M(c)$ is constant on $[c_L, \bar{c}]$, the constraint continues to hold on the entire interval $[c_L, c_H]$.

Let

$$\begin{aligned}
&\sigma(c) = w(c)\, f(c) - \frac{[F(c) + c\, f(c)]\; w(\bar{c})\, f(\bar{c})}{F(\bar{c}) + \bar{c}\, f(\bar{c})}, && \forall c \in [c_L, \bar{c}],\\
&\xi(c) = -\phi'(c), && \forall c \in [\bar{c}, c_H],\\
&\xi_T = \phi(c_H),
\end{aligned}$$

where

$$\phi(c) = \frac{1}{c^2\, f(c)} \underbrace{\left[c\, w(c)\, f(c) - \int_{\bar{c}}^{c} w(y)\, f(y)\, \mathrm{d}y - \frac{\bar{c}\, w(\bar{c})\, F(\bar{c})\, f(\bar{c})}{F(\bar{c}) + \bar{c}\, f(\bar{c})} \right]}_{\equiv N(c)}.$$

Since

$$\begin{aligned}
\xi_T \ge 0 \quad &\Leftrightarrow \quad N(c_H) \ge 0\\
&\Leftrightarrow \quad c_H\, w(c_H)\, f(c_H) - \int_{\bar{c}}^{c_H} w(y)\, f(y)\, \mathrm{d}y - \frac{\bar{c}\, w(\bar{c})\, F(\bar{c})\, f(\bar{c})}{F(\bar{c}) + \bar{c}\, f(\bar{c})} \ge 0\\
&\Leftrightarrow \quad -h(c_H) + h(\bar{c}) + \frac{\bar{c}^2\, w(\bar{c})\, f(\bar{c})^2}{F(\bar{c}) + \bar{c}\, f(\bar{c})} \ge 0,
\end{aligned}$$

where h is defined by (14), to verify the nonnegativity of ξ_T, it suffices to show that

$$-h(c_H) + h(\bar{c}) + \frac{\bar{c}^2\, w(\bar{c})\, f(\bar{c})^2}{F(\bar{c}) + \bar{c}\, f(\bar{c})} \ge 0.$$

Recall that the monotonicity of $h(c) + \frac{c^2\, w(c)\, f(c)^2}{F(c) + c\, f(c)}$ was established in the proof of Proposition 2. When $\tilde{c} < c_0 = c_H$ and $B = \tilde{B}$, the inequality above holds with equality. Since $\bar{c}$ is increasing in B, the desired result holds automatically.

When $\tilde{c} = c_H$, it suffices to show that

$$-h(c_H) + h(c_L) + c_L\, w(c_L)\, f(c_L) \ge 0,$$

which follows from

$$-h(c_H)+h(c_L)+c_L\,w(c_L)\,f(c_L)=-h(c_H)\geq 0.$$

The last inequality follows from the definition of $\tilde{c}$.

The nonnegativity of ξ is equivalent to the monotonicity of ϕ. The desired result boils down to

$$c^2\,f(c)\underbrace{N'(c)}_{=c\,\frac{\mathrm{d}}{\mathrm{d}c}(w(c)\,f(c))\leq 0}-c\,N(c)\,[2\,f(c)+c\,f'(c)]\leq 0.$$

Notice that

$$\xi_T\geq 0\quad\Leftrightarrow\quad N(c_H)\geq 0.$$

Hence, a sufficient condition is given by $2\,f(c)+c\,f'(c)\geq 0$. It remains to prove the inequality when $2\,f(c)+c\,f'(c)<0$. The expression of ϕ can be rewritten as

$$\phi(c)=\frac{1}{c^2\,f(c)}\left[\underbrace{c\,w(c)\,f(c)-(R-c)\,F(c)}_{=c\,(R-c)\,f(c)-R\,F(c)}+\underbrace{(R-\bar{c})\,F(\bar{c})-\frac{\bar{c}\,w(\bar{c})\,F(\bar{c})\,f(\bar{c})}{F(\bar{c})+\bar{c}\,f(\bar{c})}}_{=\frac{R\,F(\bar{c})^2}{F(\bar{c})+\bar{c}\,f(\bar{c})}}\right],$$

and we have

$$\phi'(c)=\frac{-2\,R\,c^2\,f(c)^2+R\left[F(c)-\frac{F(\bar{c})^2}{F(\bar{c})+\bar{c}\,f(\bar{c})}\right][2\,c\,f(c)+c^2\,f'(c)]}{c^4\,f(c)^2}.$$

The corresponding monotonicity condition is given by

$$-2\,R\,c^2\,f(c)^2+R\underbrace{\left[F(c)-\frac{F\left(\bar{c}\right)^2}{F\left(\bar{c}\right)+\bar{c}\,f\left(\bar{c}\right)}\right]}_{\geq F(c)-F(\bar{c})\geq 0}\left[2\,c\,f(c)+c^2\,f'(c)\right]\leq 0,$$

where the inequality is implied by $2\,f(c)+c\,f'(c)<0$.

The nonnegativity of σ follows from

$$\sigma(c)=w(c)\,f(c)-\frac{w(\bar{c})\,f(\bar{c})\,[F(c)+c\,f(c)]}{F(\bar{c})+\bar{c}\,f(\bar{c})}=R\left[F(\bar{c})+\bar{c}\,f(\bar{c})-\left[c+\frac{F(c)}{f(c)}\right]\,f(\bar{c})\right]\,f(c)\geq 0,$$

where the inequality follows from the monotonicity of $c+\frac{F(c)}{f(c)}$.

Because

$$\sigma(c)+[F(c)+c\,f(c)]\left[\int_{\bar{c}}^{c_H}\xi(y)\,\mathrm{d}y+\xi_T\right]=w(c)\,f(c),\qquad\forall c\in[c_L,\bar{c}],$$

$$\left[\int_{\bar{c}}^{c_H}\xi(z)\,\mathrm{d}z+\xi_T\right]F(\bar{c})+\int_{\bar{c}}^{c}\left[\int_{y}^{c_H}\xi(z)\,\mathrm{d}z+\xi_T\right]f(y)\,\mathrm{d}y+c\,f(c)\left[\int_{c}^{c_H}\xi(y)\,\mathrm{d}y+\xi_T\right]=w(c)\,f(c),\quad\forall c\in[\bar{c},c_H],$$

we have

$$\int_{c_L}^{\bar{c}}Q(c)\,\sigma(c)\,\mathrm{d}c+\int_{\bar{c}}^{c_H}\left[\int_{c_L}^{c}\left[\int_{y}^{c_H}Q(z)\,\mathrm{d}z+y\,Q(y)\right]\mathrm{d}F(y)\right]\xi(c)\,\mathrm{d}c+\xi_T\int_{c_L}^{c_H}\left[\int_{y}^{c_H}Q(z)\,\mathrm{d}z+y\,Q(y)\right]\mathrm{d}F(y)$$

$$=\int_{c_L}^{c_H}w(c)\,Q(c)\,f(c).$$

Hence, the objective function is bounded above by

$$\int_{c_L}^{\bar{c}} \sigma(c)\,\mathrm{d}c + \frac{B}{N}\int_{\bar{c}}^{c_H} \left[1-[1-F(c)]^N\right] \xi(c)\,\mathrm{d}c + \frac{B}{N}\,\xi_T.$$

Our candidate solution achieves this upper bound. By subtracting the candidate's expected objective value from the upper bound, applying integration by parts, and substituting the definitions of Q and the dual variables, the difference simplifies strictly to the boundary terms evaluated at $\bar{c}$:

$$-\frac{\bar{c}\,w(\bar{c})\,F(\bar{c})\,f(\bar{c})}{F(\bar{c})+\bar{c}\,f(\bar{c})} + \underbrace{\frac{B}{N}\left[1-[1-F(\bar{c})]^N\right]}_{=\bar{c}\left[B\int_{\bar{c}}^{c_H}\frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y+1\right]F(\bar{c})} \frac{w(\bar{c})\,f(\bar{c})}{F(\bar{c})+\bar{c}\,f(\bar{c})} - B\,\frac{\bar{c}\,w(\bar{c})\,F(\bar{c})\,f(\bar{c})}{F(\bar{c})+\bar{c}\,f(\bar{c})}\int_{\bar{c}}^{c_H}\frac{[1-F(c)]^{N-1}}{c^2}\,\mathrm{d}c = 0,$$

where the exact cancellation to zero follows directly from the threshold condition that determines $\bar{c}$.

This completes the proof. □

Proof of Corollary 3. The proof follows analogously to that of Corollary 2 and is therefore omitted. □

Proof of Proposition 4. Consider a relaxed problem that ignores all constraints except the feasibility condition on Q. Given that the virtual valuation function w is decreasing, the optimal solution to this problem is $Q(c) = \mathbb{1}_{c\le c_0}$. Under this Q, the corresponding M recovered from envelope condition (EC.4) is given by $M(c) = c_0 \cdot \mathbb{1}_{c\le c_0}$.

For any $c \in [c_L, c_0]$, we have

$$\int_{c_L}^{c} M(y)\,\mathrm{d}F(y) = c_0\,F(c),$$

and the corresponding feasibility condition is given by

$$c_0\,F(c) \le \frac{B}{N}\left[1-[1-F(c)]^N\right] \quad \Leftrightarrow \quad B \ge \frac{N\,c_0\,F(c_0)}{1-[1-F(c_0)]^N},$$

indicating that the solution to the relaxed problem remains feasible when the previously ignored constraints are reintroduced.

This establishes the optimality of the solution and thus completes the proof. □

Proof of Corollary 4. By construction, for any $c_1 \in [c_L, c_H]$, we have

$$\int_{c_{-i}\in[c_L,c_H]^{N-1}} q(c_i, c_{-i}) \prod_{k\in[N]\setminus\{i\}} \mathrm{d}F(c_k) = Q(c_i).$$

For any $c \in [c_L, c_0]$, we have

$$\begin{aligned}
\int_{c_{-i}\in[c_L,c_H]^{N-1}} m(c_i, c_{-i}) \prod_{k\in[N]\setminus\{i\}} \mathrm{d}F(c_k) &= \sum_{j=0}^{N-1} \frac{\bar{B}}{j+1}\binom{N-1}{j} F(c_0)^j\,[1-F(c_0)]^{N-1-j} \\
&= \sum_{j=0}^{N-1} \frac{\bar{B}}{j+1}\,\frac{(N-1)!}{j!\,(N-j-1)!}\,F(c_0)^j\,[1-F(c_0)]^{N-1-j} \\
&= \frac{\bar{B}}{N\,F(c_0)} \sum_{j=0}^{N-1} \frac{N!}{(j+1)!\,(N-j-1)!}\,F(c_0)^{j+1}\,[1-F(c_0)]^{N-1-j} \\
&= \frac{\bar{B}}{N\,F(c_0)} \sum_{k=1}^{N} \binom{N}{k} F(c_0)^k\,[1-F(c_0)]^{N-k} \\
&= \frac{\bar{B}}{N\,F(c_0)}\left[1-[1-F(c_0)]^N\right] \\
&= c_0.
\end{aligned}$$

For any $c \in [c_0, c_H]$, we have

$$\int_{c_{-i} \in [c_L, c_H]^{N-1}} m(c_i, c_{-i}) \prod_{k \in [N] \setminus \{i\}} \mathrm{d}F(c_k) = 0,$$

confirming that taking expectation of $m(c_i, c_{-i})$ with respect to c_{-i} yields $M(c)$.

Meanwhile, for any reporting profile $(c_i, c_{-i}) \in [c_L, c_H]^N$, we have $\sum_{i=1}^{N} m(c_i, c_{-i}) \leq B$ and $0 \leq q(c_i, c_{-i}) \leq 1$, implying its feasibility.

This completes the proof. □