Government Funding Allocation in Anti-Obesity Medicine R&D: Theory and Experiments

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As obesity remains one of the most prevalent health concerns in the United States, the government and other public sectors have increasingly invested in pharmaceutical R&D for anti-obesity medicine. Beyond maximizing expected payoff, the government has broader responsibilities, such as collecting accurate market information (e.g., private R&D costs) to support the design of effective subsidies and policy interventions for obesity treatment. However, pharmaceutical firms may misreport their private costs when competing for public funding, complicating efficient allocation. We therefore design an allocation mechanism that captures the key features of public investment in anti-obesity medicine R&D. In a static setting with a government with a limited budget and multiple firms, this mechanism incentivizes truthful reporting while maximizing the government's expected payoff. We then conduct laboratory experiments comparing our mechanism with two benchmarks: a threshold mechanism and a first-price auction-based (FPA-based) mechanism. The results show that the proposed mechanism induces more truthful reporting of private information and that its empirical performance aligns well with theoretical expectations. In contrast, the FPA-based mechanism exhibits substantial discrepancies between theory and experiment. A second experimental study suggests that risk aversion and ambiguity aversion induce agents to adopt more conservative reporting strategies under the FPA-based mechanism. Supported by both theoretical and experimental evidence, our proposed mechanism offers guidance for designing funding policies and subsidy schemes that align pharmaceutical R&D incentives with broader public health objectives.

Key words: pharmaceutical R&D, anti-obesity, mechanism design, lab experiment, behavioral operations

1. Introduction

Obesity has emerged as a major public health concern in recent years. According to data from the National Health Interview Survey (NHIS), 33.1% of respondents reported being affected by obesity in 2022, making it the most prevalent health condition in the United States that year (National Institutes of Health 2025). This trend raises two major concerns: (1) a public health crisis, as obesity is a well-established risk factor for chronic diseases such as type 2 diabetes, cardiovascular

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disease, and cancer (Stierman et al. 2021); and (2) a substantial economic burden on healthcare systems. For instance, in the United States, obesity is estimated to account for more than $170 billion annually in direct medical expenditures (Ward et al. 2021), in addition to the additional costs associated with related comorbidities.

In response to growing public health concerns, the National Institutes of Health (NIH) allocated $1.19 billion to obesity-related research and early-stage pharmaceutical development (R&D) in 2022, with funding expected to rise to $1.28 billion by 2025 (National Institutes of Health 2025). For example, through the Small Business Innovation Research (SBIR) program, the NIH awarded $295,923 to MelliCell, Inc. to develop industrial-scale technology for drug discovery in mature human adipocytes, aiming to enable the development of novel biologic drugs and therapeutic interventions (U.S. Small Business Administration 2024). Apart from the NIH, other government agencies such as the National Science Foundation (NSF) and the U.S. Department of Defense (DoD) also actively fund obesity-related research and early-stage pharmaceutical development (e.g., CDMRP 2022, Business Wire 2024). These examples echo the current urgency and necessity of developing next-generation anti-obesity medicines with enhanced efficacy. As reported by *The New York Times*, "Some new (anti-obesity) medications in development are also trying to solve for a persistent side effect of existing drugs: Patients who lose fat also tend to lose muscle. This can be particularly dangerous for older adults because it makes them more likely to fall and can worsen osteoporosis" (Kolata 2025). Therefore, for these government agencies, it is important to make efficient investments in supporting anti-obesity medicine R&D.

In academia, optimizing funding allocation in pharmaceutical R&D is also a prominent research question. For example, Hassanzadeh et al. (2014) develops a robust optimization model for contract research organizations (CROs) to optimize project selection and scheduling, accounting for uncertainty in pharmaceutical R&D costs. In another study, Park et al. (2015) designs a natural experiment to examine the NIH's project selection preferences and to analyze its strategic decisions regarding pharmaceutical R&D funding allocation. These prior studies analyze funding allocation strategies to increase profit, which is an important objective in the funding allocation problem.

However, beyond maximizing profit, designing funding strategies also involves other important objectives that have remained underexplored in prior literature. In practice, when allocating pharmaceutical R&D funding for diseases with significant public health implications, governments are also motivated to elicit truthful private information about disease characteristics and the potential pharmaceutical market.[1] Obesity is one such disease, regarded as one of the most influential pub-

[1] This is because, for emerging medicines related to public health, only pharmaceutical firms possess private information about key parameters such as R&D costs. Without access to this information, the government may misjudge the appropriate scale of funding and the prioritization of projects. Moreover, eliciting such private information can help the government design future incentive mechanisms that not only encourage firm participation but also deter firms from overstating costs to secure excessive subsidies.

lic health problems. According to a report from the World Health Organization (WHO), 43% of adults, amounting to 2.5 billion people worldwide, are overweight and require treatments at different levels of severity (World Health Organization 2024). In addition, obesity is a known risk factor for several fundamental diseases that are major public health concerns, including hypertension, high cholesterol, diabetes, cardiovascular diseases, respiratory and musculoskeletal disorders, and various forms of cancer. Therefore, understanding truthful private information about anti-obesity medicine and the pharmaceutical market is critical not only for optimizing funding strategies but also for enabling the government to fulfill its broader public health responsibilities, such as designing targeted public health interventions and policies, forecasting nationwide obesity treatment needs, identifying innovation gaps, and ensuring accountability in future investment decisions.

Based on this analysis, we design an allocation mechanism that is incentive-compatible and maximizes the government agency's expected payoff. Our design incorporates features of public investment and anti-obesity medicine R&D that differ from general pharmaceutical R&D. First, in the U.S., anti-obesity medications (AOMs) are generally excluded from Medicare Part D when used for weight loss (ASPE 2024). This limits routine Medicare claim visibility and reinforces information asymmetries between the government and firms on anti-obesity medicine. Second, governments primarily fund basic and early-stage research in obesity and other therapeutic areas (Galkina Cleary et al. 2018), providing limited direct support for late-stage clinical trials outside strategic exceptions (e.g., vaccines and pandemics). Therefore, our analysis focuses on government investment in the early stages of anti-obesity medicine R&D. Third, early-stage anti-obesity medicine R&D is dominated by small biotechs with limited track records and high technical uncertainty (Taylor 2025, Melson et al. 2025, Mylappagari and Shukla 2024). Because success probabilities vary widely by target and modality, ex ante probabilities are unreliable. We therefore model firms as holding private cost information and maximizing expected payoff rather than relying on firm-reported success probabilities.

By the revelation principle (Myerson 1981), we can focus on incentive-compatible direct mechanisms without loss of generality. Under such mechanisms, truthful reporting is optimal for a risk-neutral firm, allowing the government agency to obtain accurate information to guide funding decisions and broader policy objectives. In our study, we consider a static setting with multiple firms ($N \geq 2$), each possessing a research project with privately known development costs.[2] The government agency, operating under a limited budget, allocates funding across firms. We fully characterize the optimal mechanism in this environment under standard regularity conditions.

[2] In our study, the terms *private cost* and *true cost* refer to the same value, as the actual development costs are privately known by firms. We use these two terms contextually, depending on whether we are emphasizing an agent's subjective perspective or an objective measure.

While our theoretical mechanism assumes that agents are fully rational and self-interested, this assumption may not hold in practice. In real-world settings, agents such as small firms and startups may exhibit bounded rationality, meaning they cannot process all information perfectly or deliberate indefinitely, and may also display limited strategic reasoning or behavioral biases when interacting with complex funding mechanisms. These behavioral deviations could affect the incentive compatibility and efficiency of the mechanism. Under these circumstances, agents may not truthfully report their private information even when doing so is optimal, and thus lower the government's expected payoff.

Therefore, we propose two hypotheses to examine the performance of the mechanism while accounting for agents' deviations from rational behavior: (1) whether agents will truthfully report their private information (Hypothesis 1), and (2) whether our optimal mechanism can effectively maximize the principal's expected payoff compared to benchmark mechanisms (Hypothesis 2). The benchmark mechanisms are the threshold mechanism and the first-price auction-based (FPA-based) mechanism. The former refers to a policy in which the principal funds all projects with reported costs below a fixed threshold, providing each with an equal amount of investment. The latter adopts a first-price auction–style allocation rule, where participants report their costs, and the lowest-cost project receives funding from the government first, with the remaining budget allocated to the project with the second-lowest cost, and so on. To test these hypotheses, we conduct a laboratory experiment implementing our proposed mechanism to examine whether it induces truthful reporting and performs consistently with theoretical predictions under more realistic behavioral conditions.

Our lab experiment follows a 2×3 between-subjects experimental design. In this experiment, subjects act as agents in a simulated anti-obesity medicine R&D scenario, where, consistent with the proposed mechanism, they strategically report their private costs when interacting with a principal representing the government. We recruited 396 participants via Amazon Mechanical Turk. To ensure data quality and engagement, we incorporated an attention check to exclude inattentive participants and implemented an incentive scheme that awarded bonuses to those whose performance exceeded that of their peers.

In line with Hypothesis 1, we experimentally find that under the optimal mechanism, there is no significant deviation between their reported and true private costs. Moreover, compared with the threshold and FPA-based benchmark mechanisms, the optimal mechanism yields smaller reporting deviations, indicating that it more effectively incentivizes truthful reporting. However, the results also show that while the optimal mechanism generally generates higher returns than the threshold mechanism, it performs similarly to or worse than the FPA-based mechanism across different budget conditions, thereby failing to support Hypothesis 2.

As our proposed optimal mechanism performs consistently across theory and experiments, the observed discrepancies in comparison arise from the FPA-based mechanism. Under this mechanism, agents adopt a conservative reporting strategy, slightly shading down their reported costs to secure a safer position, even at the expense of reducing their own profits. To further investigate the behavioral explanations for this conservative reporting strategy, we draw on the theories of risk aversion (Pratt 1978) and ambiguity aversion (Ellsberg 1961) and design a second laboratory experiment to examine whether these behavioral factors drive the observed performance biases of the FPA-based mechanism. We adopt a 2×2 experimental design that exogenously manipulates the salience of risk aversion (high vs. low) and ambiguity aversion (high vs. low), creating four experimental groups representing all combinations of the two factors. The logic of the experiment is as follows: if, after exogenously increasing the salience of risk aversion or ambiguity aversion, participants in the treatment groups report significantly lower private costs than those in the corresponding control groups (where salience is not enhanced), this would indicate that risk aversion or ambiguity aversion is an underlying factor that leads participants to report their private costs more conservatively. The experimental procedure and rules are similar to those in the first experiment, except for the priming manipulation. The experimental results show that both risk aversion and ambiguity aversion lead agents to report their private costs in an irrationally conservative manner under the FPA-based mechanism, thereby resulting in the principal's payoff under the FPA-based mechanism performing similarly to or slightly better than that under our proposed optimal mechanism.

Our study makes two main contributions. First, we design a funding allocation mechanism for governments that accounts for the characteristics of anti-obesity medicine R&D. This mechanism enables government agencies to meet the dual objectives of eliciting firms' private cost information and ensuring the government's expected payoff in theory. Second, we conduct lab experiments to empirically study behavioral deviations from rationality in the funding allocation process, provide behavioral explanations for these deviations, and demonstrate the practical usefulness of our mechanism. To the best of our knowledge, this study is the first to propose a mechanism-based approach, grounded in both theoretical analysis and empirical evidence, that enables government agencies to elicit truthful information from firms in the context of pharmaceutical R&D funding allocation. This information can, in turn, support broader public health objectives, such as designing targeted health interventions, forecasting national treatment needs, and informing future investment decisions in this area.

The remainder of this paper is organized as follows: §2 summarizes the related literature; §3 discusses our problem and introduces the model; §4 characterizes the optimal funding mechanism; §5 develops the hypotheses regarding the effectiveness of our mechanism under realistic behavioral conditions and details our lab experiment for empirical testing; §6 presents the empirical results and discusses behavioral deviations from rationality; and §7 concludes the paper.

2. Literature Review

Our paper is related to the existing literature on (1) resource allocation of public sectors, (2) pharmaceutical R&D, (3) mechanism design with budget constraints, and (4) behavioral operations in decision-making. In this section, we review the relevant literature and outline the contributions of our work.

The resource allocation problem in the public sector has been extensively studied in prior literature. Because public agencies carry significant social responsibilities, revenue maximization is not their primary objective when allocating limited resources. Instead, they focus on promoting social welfare, such as reducing wealth inequality, providing fundamental support for disadvantaged populations and resource-limited regions, and fostering societal business innovation. For example, Tingley and Liebman (1984) formulates a linear integer optimization problem to support state-level resource allocation in the U.S. Department of Agriculture's Special Supplemental Nutrition Program for Women, Infants, and Children (WIC). Lien et al. (2014) examines the problem of nonprofit organizations distributing a scarce resource to meet customer demands that arrive sequentially, using a dynamic programming framework. Roet-Green and Shetty (2022) studies the resource allocation problem faced by a welfare-maximizing service provider who must distribute a fixed quantity of resources between two service variants, a standard option and an expedited option, with an application to U.S. airports through the TSA PreCheck program. Singh and Wu (2025) explores the efficient allocation of a divisible resource, such as water in water-scarce regions, and proposes mechanisms to maximize aggregate consumer welfare, accounting for consumers' heterogeneous incomes and private valuations of the resource.

In the context of R&D, prior studies have examined how public sector institutions can enhance R&D efficiency and promote societal business innovation. For instance, Bruce et al. (2019) studies U.S. federal R&D contracts, which fall into two categories: grants, which involve minimal oversight, and cooperative agreements, which grant decision rights during the project. They show that cooperative agreements can improve efficiency for early-stage, high-uncertainty projects, particularly when government scientists with relevant expertise are located near the firm's R&D site. Similarly, Gao et al. (2022) explores optimal resource allocation strategies for both nonprofit and profit-maximizing principals who must allocate limited resources to support innovations by multiple, potentially competing, innovators. A recurring challenge in R&D funding allocation is the presence of private information held by agents, which can undermine allocation efficiency and ultimately harm social welfare (Esteban and Ray 2006, Singh and Wu 2025). To address this issue, recent research highlights the importance of designing mechanisms that incentivize agents to truthfully report their private information. Truthful reporting enables better decision-making and can lead to improved social outcomes, particularly in public sector R&D efforts. Our study also highlights the importance of designing mechanisms that incentivize agents to truthfully report their private information in the pharmaceutical R&D setting.

While general R&D allocation studies provide broad frameworks applicable across industries, a growing stream of research focuses specifically on pharmaceutical R&D, addressing the unique challenges of drug development and offering more industry-specific insights. For example, Vernon (2005) studies the adverse impact of price regulation on pharmaceutical R&D investment. Chan et al. (2007) develops a dynamic programming model to explain why firms adopt time-varying strategic thresholds when selecting R&D projects. Ganuza et al. (2009) analyzes the incentives for pursuing minor improvements to existing compounds and finds that small innovations often receive disproportionately high rewards, as firms tend to target inelastic segments of demand. Rao (2020) builds a dynamic investment model to analyze the strategic decisions of a pharmaceutical firm in response to its competitors in drug R&D, and estimates structural parameters using data from phase III trials. Krieger et al. (2022) investigates how firms' investment decisions are affected by negative shocks to existing products, using FDA public health advisories as exogenous shocks to approved drugs. Our study contributes to this stream of literature by proposing a theoretical mechanism that considers the characteristics of anti-obesity medicine R&D and by validating its practical performance through experimental studies. It aims both to elicit truthful private cost information from firms and to maximize the expected payoff of governments.

Our theoretical analysis of the funding allocation problem is grounded in the literature on auctions with financial constraints, where a principal must allocate scarce resources among privately informed agents. Laffont and Robert (1996) introduces one of the earliest models of optimal auctions with buyers subject to a common budget constraint, showing how limited liquidity distorts allocation relative to standard auctions. Maskin (2000) extends this framework by assuming that the common budget level is exogenous and commonly known, and analyzes the implications for allocative efficiency. Malakhov and Vohra (2008) derives the revenue-maximizing auction in an asymmetric setting with two budget-constrained bidders. Che and Gale (2000) studies a seller's problem where a single buyer has private information about both valuation and budget, which may be correlated. The optimal mechanism departs from standard monopoly pricing and might involve a nonlinear pricing scheme. Benoit and Krishna (2001) considers sequential and simultaneous auctions of multiple objects with common values and budget constraints, and also allows for endogenous budget levels. Pai and Vohra (2014) extends the setting to one in which both valuations and budget levels are private information, and they derive the symmetric revenue-maximizing and constrained-efficient auctions. Key distinctions in our investment problem are that the sum of monetary transfers to agents is bounded, rather than the charge to each bidder as in these problems, and that the quantity of the good being sold is not restricted to one. In other words, the funder can allocate resources to multiple firms, with each share not exceeding one. These features make our problem mathematically distinct.

Finally, our study is closely related to the stream of literature on behavioral biases in decision-making. Theoretical analysis, particularly those based on utility-based models, typically assumes rational human behavior. However, behavioral biases in decision-making inevitably exist, and practical performance can therefore deviate from optimality, reducing the effectiveness of theoretical approaches (Donohue et al. 2019). For this reason, an increasing number of studies have begun to investigate behavioral biases in decision-making. For example, Beer et al. (2025) shows that workers often deviate significantly from the optimal policy, either by taking longer than normative theory suggests, failing to complete their tasks, or experiencing greater delays when collaborating with robots. Batt and Tong (2020) investigates how server-level service-quality metrics can diverge from customer-experienced metrics and how these discrepancies bias human judgment and decision-making. Beil et al. (2024) finds that behavioral biases, including mis-weighting future periods and focusing only on high-value opportunities, can impair resource allocation decisions of product managers in complex project settings.

In particular, some studies focus on behavioral factors in the healthcare context to improve operational performance. For example, Kim et al. (2020) analyzes the cognitive and environmental factors that drive systematic admission decision biases in a simulated hospital unit. Bavafa and Jónasson (2021) investigates behavioral variability, specifically how performance consistency improves with experience, and highlights important implications for staffing efficiency and service reliability in healthcare delivery. Liang et al. (2024) conducts field and lab experiments to examine discretionary pricing by pharmaceutical store managers, focusing on price increases for high-priced drugs and the behavioral factors behind them. We contribute to this stream of literature by experimentally examining behavioral deviations under a designed funding allocation mechanism. Our study provides implications for government investment in drug R&D by testing whether the theoretical mechanism effectively motivates agents to reveal truthful private information in practice, enabling policymakers to allocate resources more efficiently, reduce informational asymmetries, and design funding policies that align private incentives with public goals.

3. Model

Consider a setting in which a government agency (the principal, hereafter the "funder") interacts with $N \geq 2$ agents, indexed by $i \in [N] \equiv \{1, \ldots, N\}$, in a static game. Each agent represents a researcher or start-up (hereafter a "firm") undertaking an early-stage project that constitutes one step in the broader drug development process. All participants are risk-neutral. Each project yields an expected societal value of R to the funder, where R can be interpreted as the product of the project's success probability and its value conditional on success. Firm i's cost of development, denoted c_i, is independently and identically drawn from a distribution with support $[c_L, c_H]$, cumulative distribution function F, and density f. The realization of c_i is privately known to the firm.

We model a firm's private information as its development cost rather than its probability of success, because at this early stage, startups and small firms typically lack reliable estimates of their success probabilities due to limited track records and high technical uncertainty. This setup is equivalent to one in which the private information is the project's return, given by R/c_i. Since the funder has no incentive to support firms with $c_i > R$, we assume without loss of generality that $c_H \leq R$. The funder has a limited budget B and aims to maximize its expected payoff, defined as the expected social value of funded projects net of total transfers, by allocating funding across the N firms.

By the revelation principle (Myerson 1981), we can restrict attention to direct symmetric incentive-compatible mechanisms.[3] That is, given the reported cost vector $\mathbf{c} \in [c_L, c_H]^N$, the mechanism specifies: (1) an "allocation" rule $q(c_i, c_{-i})$, denoting the share of firm i's project claimed by the funder; (2) a monetary transfer $m(c_i, c_{-i})$ from the funder to firm i. Here, c_i denotes firm i's reported type, and $c_{-i} \equiv (c_1, c_2, \ldots, c_{i-1}, c_{i+1}, \ldots, c_N)$ denotes the vector of reported types of all its competitors. We allow firms to sell their remaining project share to outside investors at marginal cost, and hence, the firm's payoff from its outside option is normalized to 0.

Let $\hat{u}(c, \hat{c})$ denote firm i's *expected* payoff when its true cost is c but it reports $\hat{c}$. As specified by the mechanism, upon observing the reported types of its competitors, $c_{-i} \in [c_L, c_H]^{N-1}$, the funder makes a monetary transfer $m(\hat{c}, c_{-i})$ to firm i and claims a share $q(\hat{c}, c_{-i})$ of the project, which imposes an expected cost of $c\,q(\hat{c}, c_{-i})$ on the firm. Because early-stage projects yield no immediate revenue to the firm, its expected payoff equals the monetary transfer minus its share of the development cost, i.e.,

$$\hat{u}(c, \hat{c}) = \int_{[c_L, c_H]^{N-1}} [m(\hat{c}, c_{-i}) - c\,q(\hat{c}, c_{-i})] \prod_{k \in [N] \setminus \{i\}} \mathrm{d}F(c_k), \quad \forall (c, \hat{c}) \in [c_L, c_H]^2. \tag{1}$$

In this scenario, the funder "purchases" a share $q(\hat{c}, c_{-i})$ of firm i's project at a cost of $m(\hat{c}, c_{-i})$. In equilibrium, all the firms have the incentive to report their costs truthfully. Let U denote the funder's expected payoff, and we have

$$U \equiv N \int_{[c_L, c_H]^N} [R\,q(c_i, c_{-i}) - m(c_i, c_{-i})] \prod_{i=1}^{N} \mathrm{d}F(c_i). \tag{2}$$

[3] The symmetry follows from footnote 11 in Maskin and Riley (1984).

A feasible mechanism must satisfy the incentive-compatible constraint, meaning that a firm with cost c should not benefit from misreporting its type as $\hat{c}$, i.e.,

$$\hat{u}(c,c) \geq \hat{u}(c,\hat{c}), \quad \forall (c,\hat{c}) \in [c_L, c_H]^2. \tag{IC}$$

Furthermore, since the value of the firm's outside option is normalized to 0, the following individual rationality constraint must hold to guarantee the firm's participation:

$$\hat{u}(c,c) \geq 0, \quad \forall c \in [c_L, c_H]. \tag{IR}$$

Beyond the standard (IC) and (IR) constraints, since the funder has only a limited budget B, the total amount allocated among all agents cannot exceed this limit, i.e.,

$$\sum_{i=1}^{N} m(c_i, c_{-i}) \leq B, \quad \forall (c_i, c_{-i}) \in [c_L, c_H]^N. \tag{3}$$

Finally, the share claimed by the funder from each firm must be in $[0,1]$, i.e.,

$$0 \leq q(c_i, c_{-i}) \leq 1, \quad \forall (c_i, c_{-i}) \in [c_L, c_H]^N, \tag{4}$$

and we impose the following limited liability constraint to prevent the funder from charging the firm, i.e.,

$$m(c_i, c_{-i}) \geq 0, \quad \forall (c_i, c_{-i}) \in [c_L, c_H]^N. \tag{5}$$

To summarize, the funder's problem can be formulated as follows:

$$\max_{\{q(\cdot), m(\cdot)\} \in \Omega_0} \ U, \tag{6}$$

where the feasible set Ω_0 is determined by linear constraints (IC), (IR), (3), (4), and (5).

The optimization problem (6) involves two N-dimensional functions, making it technically challenging to solve directly. To simplify the analysis, we introduce the following *interim* allocation and payment functions

$$Q(c_i) \equiv \mathbb{E}_{c_{-i}}\left[q(c_i, c_{-i})\right] = \int_{[c_L, c_H]^{N-1}} q(c_i, c_{-i}) \prod_{k \in [N] \setminus \{i\}} \mathrm{d}F(c_k), \quad \forall c_i \in [c_L, c_H] \tag{7}$$

and

$$M(c_i) \equiv \mathbb{E}_{c_{-i}}\left[m(c_i, c_{-i})\right] = \int_{[c_L, c_H]^{N-1}} m(c_i, c_{-i}) \prod_{k \in [N] \setminus \{i\}} \mathrm{d}F(c_k), \quad \forall c_i \in [c_L, c_H]. \tag{8}$$

The functions Q and M represent the firm's expected share claimed by, and expected payment received from, the funder, respectively, when it reports its cost as c_i, assuming all other firms report their private information truthfully. Both parties' payoff functions, defined by (1) and (2), can be rewritten as

$$\hat{u}(c,\hat{c}) = M(\hat{c}) - c\,Q(\hat{c}), \quad \forall (c,\hat{c}) \in [c_L, c_H]^2$$

and

$$U = N \int_{c_L}^{c_H} [R\,Q(c) - M(c)]\,\mathrm{d}F(c).$$

Border (1991) proves that the budget constraint (3) holds if and only if[4]

$$\int_{c_L}^{c} M(y)\,\mathrm{d}F(y) \le \frac{B}{N}\left[1 - [1-F(c)]^N\right], \quad \forall c \in [c_L, c_H]. \tag{9}$$

The remaining feasibility constraints (4) and (5) boil down to

$$0 \le Q(c) \le 1, \quad \forall c \in [c_L, c_H], \text{ and} \tag{10}$$

$$M(c) \ge 0, \quad \forall c \in [c_L, c_H], \tag{11}$$

respectively. The funder's problem can be reformulated as follows:

$$\max_{\{Q(\cdot), M(\cdot)\} \in \Omega} U, \tag{12}$$

where the feasible set Ω is determined by linear constraints (IC), (IR), (9), (10), and (11).

4. Optimal Mechanism

In this section, we characterize the optimal mechanism that solves the funder's problem (12) and the corresponding implementation. We make the following assumption throughout the paper.

Assumption 1. *The density function f is non-increasing.*

Assumption 1 is standard in the mechanism design literature. It implies that the inverse hazard ratio, $F(c)/f(c)$, is increasing, and it holds for several commonly used distributions, including the uniform, truncated exponential, and Pareto distributions.

The structure of the optimal mechanism is highly sensitive to the funder's budget B. Proposition 1 analyzes the case in which the funder's budget is limited. We defer all proofs to Appendix EC.4.

Proposition 1. *When $B \le \underline{B}$, the optimal reduced-form mechanism is given by*

$$Q(c) = \begin{cases} B\left[\frac{[1-F(c)]^{N-1}}{c} - \int_c^{\tilde{c}} \frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y\right], & \forall c \in [c_L, \tilde{c}], \\ 0, & \forall c \in (\tilde{c}, c_H], \end{cases} \qquad M(c) = \begin{cases} B\,[1-F(c)]^{N-1}, & \forall c \in [c_L, \tilde{c}], \\ 0, & \forall c \in (\tilde{c}, c_H], \end{cases} \tag{13}$$

[4] This reformulation requires the monotonicity of the interim allocation rule Q, which is standard in mechanism design problems (see Lemma EC.1).

where the thresholds $\tilde{c}$ *and* $\underline{B}$ *are defined by*

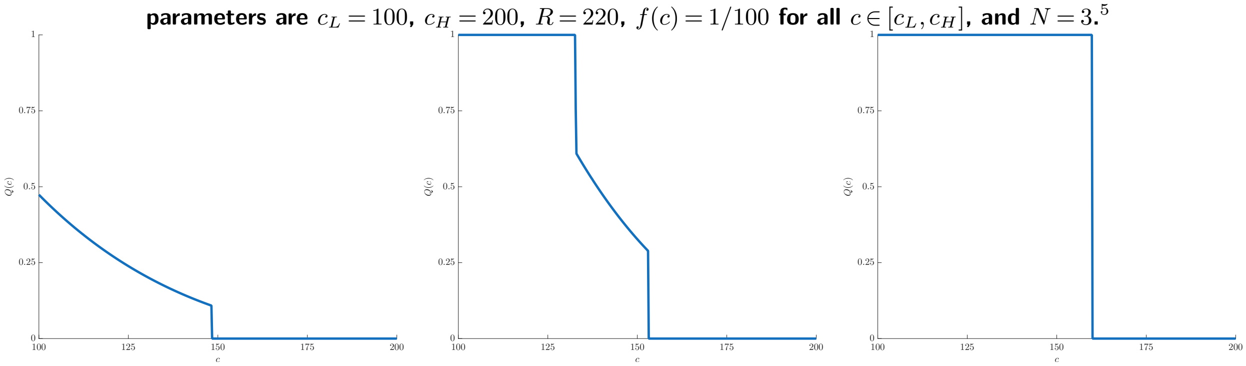
$$\tilde{c} \equiv \sup\left\{c \in [c_L, c_H] : -c\,w(c)\,f(c) + \int_{c_L}^{c} w(y)\,f(y)\,\mathrm{d}y \le 0\right\} \tag{14}$$

and

$$\underline{B} \equiv \left[\frac{1}{c_L} - \int_{c_L}^{\tilde{c}} \frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y\right]^{-1}, \tag{15}$$

respectively.

Figure 1 The optimal interim allocation rule Q specified in Propositions 1, 2, and 4, respectively. The model parameters are $c_L = 100$, $c_H = 200$, $R = 220$, $f(c) = 1/100$ for all $c \in [c_L, c_H]$, and $N = 3$.[5]



(a) $B = 60$ (b) $B = 200$ (c) $B = 500$

In this scenario, the funder's budget is limited, and the interim allocation rule Q remains strictly below 1 (see Figure 1(a)). This implies that the funder does not claim the entire project, even for firms with the lowest cost (or, equivalently, the highest return). The following corollary provides an implementation rule for the reduced-form mechanism defined in (13).

Corollary 1. *The reduced-form mechanism defined by* (13) *can be implemented as follows: for any* $(c_i, c_{-i}) \in [c_L, c_H]^N$,

$$q(c_i, c_{-i}) = Q(c_i), \quad m(c_i, c_{-i}) = \begin{cases} B, & \textit{if } c_i < \min\limits_{j \in [N]\setminus\{i\}} c_j \textit{ and } c_i \le \tilde{c}, \\ 0, & \textit{otherwise.} \end{cases} \tag{16}$$

Since the probability that two or more firms report the same minimal cost is zero, without loss of generality, we omit a tie-breaking rule and allocate no funding in such cases. This convention applies throughout the remainder of this section.

The implementation rule proposed in Corollary 1 is as follows: if a firm reports a cost c below the threshold $\tilde{c}$, the funder claims a share $Q(c)$ from it, where $Q(\cdot)$ is a decreasing function. However,

[5] Given the model parameters, which are used in the experimental setting in Sections 5 and 6, the optimality conditions in Proposition 3 are never satisfied, and thus this case does not arise.

a monetary transfer occurs only if the firm reports the lowest cost among all participants and that cost is below $\tilde{c}$. In that case, the funder allocates the entire budget B to the firm. Figure A1 in Appendix EC.3 visualizes the implementation rule when $N=2$.

As the funder's budget increases, it claims the entire project from firms with sufficiently low costs. However, for firms with higher costs, the funder continues to claim a share strictly less than 1. This behavior is formally characterized in the following proposition.

Proposition 2. *Define*

$$c_0 \equiv \sup\left\{c \in [c_L, c_H] : R - c - \frac{F(c)}{f(c)} \geq 0\right\}. \tag{17}$$

When $\tilde{c} < c_H$, $B > \underline{B}$, *and one of the following conditions holds:*

1. $c_0 < c_H$ *and* $B < \bar{B} \equiv \frac{N\, c_0\, F(c_0)}{1-[1-F(c_0)]^N}$*; or*
2. $c_0 = c_H$ *and* $B < \tilde{B}$*, where*

$$\tilde{B} = \frac{\tilde{c}_1\, F(\tilde{c}_1)}{\frac{1}{N}[1-[1-F(\tilde{c}_1)]^N] - \tilde{c}_1\, F(\tilde{c}_1) \int_{\tilde{c}_1}^{c_H} \frac{[1-F(y)]^{N-1}}{y^2}\, \mathrm{d}y}$$

and $\tilde{c}_1$ *is defined as*

$$\tilde{c}_1 \in \inf\left\{c \in [c_L, c_H] : h(c) + \frac{c^2\, w(c)\, f(c)^2}{F(c) + c\, f(c)} = h(c_H)\right\},$$

the optimal reduced-form mechanism is given as follows:

$$Q(c) = \begin{cases} 1, & \forall c \in [c_L, \bar{c}], \\ B\left[\frac{[1-F(c)]^{N-1}}{c} - \int_c^{\check{c}} \frac{[1-F(y)]^{N-1}}{y^2}\, \mathrm{d}y\right], & \forall c \in (\bar{c}, \check{c}], \\ 0, & \forall c \in (\check{c}, c_H], \end{cases} \tag{18}$$

and

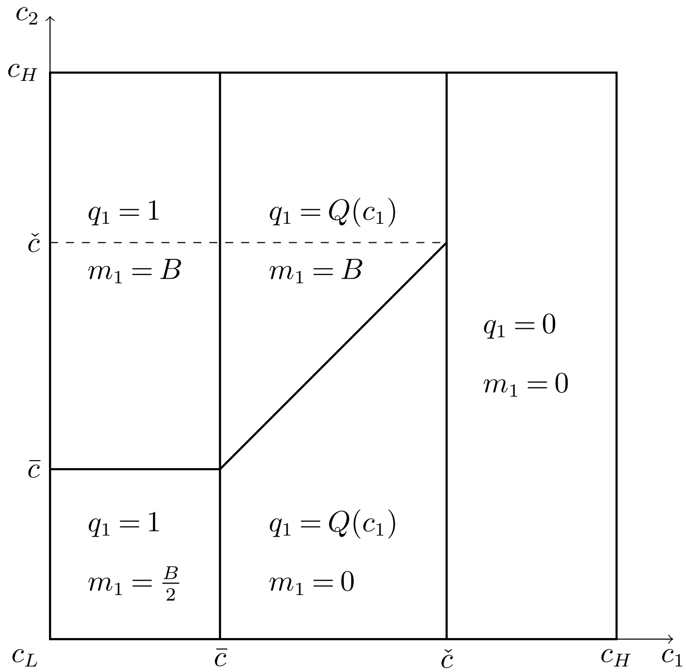
$$M(c) = \begin{cases} \bar{c}\left[B \int_{\bar{c}}^{\check{c}} \frac{[1-F(y)]^{N-1}}{y^2}\, \mathrm{d}y + 1\right], & \forall c \in [c_L, \bar{c}], \\ B\,[1-F(c)]^{N-1}, & \forall c \in (\bar{c}, \check{c}], \\ 0, & \forall c \in (\check{c}, c_H], \end{cases} \tag{19}$$

where the two thresholds $(\bar{c}, \check{c})$ *satisfy*

$$\begin{cases} [F(\bar{c}) + \bar{c}\, f(\bar{c})]\left[\bar{c}\, w(\bar{c})\, f(\bar{c}) - \check{c}\, w(\check{c})\, f(\check{c}) + \int_{\bar{c}}^{\check{c}} w(y)\, f(y)\, \mathrm{d}y\right] = \bar{c}^2\, w(\bar{c})\, f(\bar{c})^2, \\ \bar{c}\left[B \int_{\bar{c}}^{\check{c}} \frac{[1-F(y)]^{N-1}}{y^2}\, \mathrm{d}y + 1\right] F(\bar{c}) = \frac{B}{N}\left[1 - [1-F(\bar{c})]^N\right]. \end{cases} \tag{20}$$

When the funder's budget falls within the range specified in Proposition 2, we propose the implementation rule described in Corollary 2. The mechanism is governed by two thresholds, $\bar{c}$ and $\check{c}$, determined by the system of equations in (20). If firm i reports a cost below $\bar{c}$, it shares the total budget B equally with all firms submitting similarly low-cost reports, and the funder claims the entire project from it. If the reported cost lies between $\bar{c}$ and $\check{c}$, the funder claims a share strictly less than 1, and firm i receives the entire budget B only if no competitor reports a lower cost.

Figure 2 **Illustration of the implementation rule defined in Corollary 2 for the case $N=2$. We use $q_1 \equiv q(c_1,c_2)$ and $m_1 \equiv m(c_1,c_2)$ to denote the allocation and payment associated with firm 1. Since the mechanism is symmetric, the implementation for firm 2 is omitted for simplicity.**



In this intermediate range, the firm's expected payoff remains nonnegative, although its ex post payoff will be negative if it fails to secure funding. Finally, if the reported cost exceeds $\check{c}$, the firm receives no funding, and the funder claims nothing. Figure 2 depicts the ex post allocation and compensation scheme when there are two agents; the x- and y-axes correspond to the reported costs of firms 1 and 2, respectively.

Corollary 2. *The reduced-form mechanism defined by* (18) *and* (19) *can be implemented as follows: for any* $(c_i, c_{-i}) \in [c_L, c_H]^N$,

$$q(c_i, c_{-i}) = Q(c_i), \quad m(c_i, c_{-i}) = \begin{cases} B/|S|, & \text{if } c_i \le \bar{c}, \\ B, & \text{if } c_i < \min_{j \in [N]\setminus\{i\}} c_j \text{ and } \bar{c} < c_i \le \check{c}, \\ 0, & \text{otherwise}, \end{cases} \tag{21}$$

where $S \equiv \{k \in [N] : c_k \le \bar{c}\}$, *and* $|\cdot|$ *denotes the cardinality of a set.*

As the funder's budget continues to increase, it claims a strictly positive share from all firms in expectation, that is, $Q(c) > 0$ for all $c \in [c_L, c_H]$. The corresponding optimal mechanism is presented in the following proposition.

Proposition 3. *When one of the following conditions holds:*

1. $\tilde{c} < c_0 = c_H$ *and* $\tilde{B} < B \le \bar{B} = N\, c_H$*; or*
2. $\tilde{c} = c_H$ *and* $\underline{B} < B \le \bar{B} = N\, c_H$*,*

the optimal reduced-form mechanism is given as follows:

$$Q(c)=\begin{cases}1, & \forall c\in[c_L,\bar c],\\ B\left[\frac{[1-F(c)]^{N-1}}{c}-\int_c^{c_H}\frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y\right], & \forall c\in(\bar c,c_H],\end{cases}\tag{22}$$

and

$$M(c)=\begin{cases}\bar c\left[B\int_{\bar c}^{c_H}\frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y+1\right], & \forall c\in[c_L,\bar c],\\ B\left[1-F(c)\right]^{N-1}, & \forall c\in(\bar c,c_H],\end{cases}\tag{23}$$

where the threshold $\bar c$ is determined by

$$\bar c\left[B\int_{\bar c}^{c_H}\frac{\left[1-F(y)\right]^{N-1}}{y^2}\,\mathrm{d}y+1\right]F(\bar c)=\frac{B}{N}\left[1-\left[1-F(\bar c)\right]^N\right].\tag{24}$$

The implementation of the reduced-form mechanism in this case closely follows that of the previous setting (see Figure A2 in Appendix EC.3 for the case $N=2$). In this setting, we have $\check c=c_H$. If a firm reports a cost below $\bar c$, it shares the total budget B equally with all competitors who also report below $\bar c$, and the funder claims the entire project. Otherwise, the funder claims a share strictly less than one, and a monetary transfer occurs only if the firm's report is the lowest among all. The formal implementation rule is summarized in Corollary 3.

Corollary 3. *The reduced-form mechanism defined by* (18) *and* (19) *can be implemented as follows: for any $(c_i,c_{-i})\in[c_L,c_H]^N$,*

$$q(c_i,c_{-i})=Q(c_i),\quad m(c_i,c_{-i})=\begin{cases}B/|S|, & \text{if } c_i\le\bar c,\\ B, & \text{if } c_i<\min_{j\in[N]\setminus\{i\}}c_j \text{ and } c_i>\bar c,\end{cases}\tag{25}$$

where $S\equiv\{k\in[N]:c_k\le\bar c\}$, and $|\cdot|$ denotes the cardinality of a set.

Finally, when the funder's budget exceeds the level specified in Proposition 3, it is never fully exhausted ex post. In this case, the budget constraint is not binding, and the funder claims the entire project from firms whose reported costs fall below a certain threshold. Payments are designed to ensure that the threshold type is indifferent between participating and opting out. The corresponding optimality conditions are presented in the following proposition.

Proposition 4. *When $B\ge\bar B=\frac{N\,c_0\,F(c_0)}{1-[1-F(c_0)]^N}$, the optimal reduced-form mechanism is given as follows:*

$$Q(c)=\begin{cases}1, \ \forall c\in[c_L,c_0],\\ 0, \ \forall c\in(c_0,c_H],\end{cases}\qquad M(c)=\begin{cases}c_0, \ \forall c\in[c_L,c_0],\\ 0, \ \forall c\in(c_0,c_H],\end{cases}\tag{26}$$

where the threshold c_0 is defined by (17)*.*

The implementation rule in this setting is straightforward: if firm i reports a cost below the threshold c_0, the funder claims the entire project, and the firm shares a budget of $\bar B$ equally with all competitors who also report costs below c_0. A graphical illustration for the case $N=2$ is given by Figure A3 in Appendix EC.3.

Corollary 4. *The reduced-form mechanism defined by* (26) *can be implemented as follows: for any* $(c_i, c_{-i}) \in [c_L, c_H]^N$,

$$q(c_i, c_{-i}) = Q(c_i), \quad m(c_i, c_{-i}) = \begin{cases} \bar{B}/|S|, & \text{if } c_i \leq c_0, \\ 0, & \text{otherwise}, \end{cases} \tag{27}$$

where $S \equiv \{k \in [N] : c_k \leq c_0\}$, *and* $|\cdot|$ *denotes the cardinality of a set.*

In the remainder of the paper, we refer to the mechanisms defined in Propositions 1–4 as "the optimal mechanism."

5. Hypotheses and Experiments

Our theoretical analysis provides the optimal solution for the funder's allocation problem in obesity pharmaceutical R&D. However, agents (firms) do not always make rational decisions in practice, leading to behavioral biases in decision-making. To evaluate the real-world applicability of our theoretical solution, we propose two hypotheses and test them using an incentivized lab experiment in which MTurk workers simulate firms facing different allocation mechanisms and budget conditions, mirroring the setting in Section 3.

5.1. Hypothesis Development

Although the optimal mechanisms proposed in Propositions 1 to 4 theoretically incentivize agents to truthfully report their private costs, real-world behavior may deviate from this optimal outcome due to several cognitive biases.

First, agents are not fully rational because of cognitive limitations, time constraints, and imperfect information. As a result, when mechanisms are overly complex (e.g., involving infinitely many menus), agents may be unable to process all the information required to make optimal decisions, even when the mechanism is designed to induce truth-telling. This behavior is consistent with the theory of bounded rationality (Simon 1955, 1979, Camerer et al. 2004b). Second, agents may form incorrect beliefs about how a mechanism works and then make strategic decisions based on those misbeliefs. Even when mechanisms are designed to incentivize truthful behavior, such misbeliefs can lead to systematic deviations from optimal strategies. Under these circumstances, agents may mistakenly believe that inflating their reports is beneficial. This explanation is supported by the literature on strategic misbeliefs (Kagel and Roth 2000, Costa-Gomes et al. 2001, Camerer et al. 2004a). Third, agents may distrust the reporting behavior of their competitors. They may believe that if others systematically misreport their costs, then reporting truthfully would place them at a disadvantage, thereby discouraging honest reporting. Such behavioral patterns are supported by the literature on misbeliefs about others' strategic behavior (Akerlof 1970, Rees-Jones and Skowronek 2018, McCannon and Minuci 2020). These behavioral biases can undermine the effectiveness of our proposed theoretical mechanism in practice.

To evaluate our proposed mechanism, we compare it against two benchmarks: a threshold mechanism and an FPA-based mechanism. The threshold mechanism funds all projects with reported costs below a fixed threshold, providing each with an equal investment amount. Under the FPA-based mechanism, agents report their costs, and the lowest-cost project receives funding from the government first, with the remaining budget allocated to the project with the second-lowest cost, and so on.

Given our proposed and benchmark mechanisms, we expect the degree of deviation from theoretical outcomes to differ across them. Notable studies are showing that experimental outcomes often align closely with theoretical principles for incentive-compatible mechanisms. For instance, research on storable votes in laboratory settings finds that, although participants may rely on heuristics rather than theoretically optimal strategies, the resulting payoffs and allocation outcomes are very similar to those derived from the theoretical model (Casella et al. 2006). Similarly, experimental work on VCG-type public goods mechanisms has shown that when mechanisms are transparent and easy to understand, participants tend to behave in ways that are consistent with the mechanism's intended design (Healy 2006, Chen and Ledyard 2010). Although agents may not always fully understand theoretical mechanisms within a limited time frame, we attempted to mitigate this by providing detailed instructions and examples of our optimal mechanism to facilitate comprehension. Given our efforts to reduce the cognitive burden, along with the supporting findings, we believe that the reporting behavior observed in the lab is unlikely to deviate substantially from the theoretical design.[6]

Under the threshold mechanism, firms that report costs below a predetermined threshold receive an equal share of the budget (see Appendix EC.1.2 for details). Agents aiming to maximize their expected payoff will report a cost below the threshold when their true cost falls below it. If their true cost exceeds the threshold, they can avoid a loss by reporting a very high value and effectively opting out of the bidding process. As a result, the threshold mechanism may induce substantial discrepancies between reported and true costs. Under the FPA-based mechanism, agents are paid based on their reported costs, so truthful reporting yields a zero expected payoff (see Appendix EC.1.3 for details). Therefore, in equilibrium, agents tend to over-report their costs. Compared to the FPA-based mechanism and the threshold mechanism, we expect our optimal mechanism to induce smaller deviations between agents' reported and true costs. Based on the above analysis, we formalize these expectations in Hypothesis 1.

Hypothesis 1 (Reported Costs)

[6] We also include an attention check using a simple question, such as selecting the largest number from a set of four, to ensure that participants make their decisions with logical reasoning.

1.1 Under our optimal mechanism, there is no significant difference between agents' reported costs and their true costs.

1.2 Compared to the other two benchmark mechanisms (i.e., the threshold mechanism and the FPA-based mechanism), our optimal mechanism leads to smaller deviations between agents' reported and true costs.

From a mechanism design perspective, our model ensures that truthful reporting is an incentive-compatible strategy for agents. This property allows the principal to allocate resources efficiently and achieve the optimal return. Therefore, as long as behavioral deviations from rationality are limited, our mechanism is theoretically superior to the benchmark mechanisms in maximizing the principal's expected payoff. We formalize this implication in Hypothesis 2.

Hypothesis 2 (Principal's Expected Payoff) *Compared to the other two mechanisms (i.e., the threshold mechanism and the FPA-based mechanism), our optimal mechanism generates higher expected payoffs for the principal.*

5.2. Experimental Design

In our study, we use a 2×3 between-subjects experimental design to test our main hypotheses. In this lab experiment, subjects act as agents in a simulated anti-obesity medicine R&D setting. They need to strategically disclose their costs to the principal, mirroring the setup in Section 3.

5.2.1. Participants. A total of 420 participants were recruited via Amazon Mechanical Turk to take part in our online lab experiment. Each participant received a $0.80 participation fee. Participants whose final return exceeded that of both competitors earned an additional $1.20 bonus and were entered into a lottery for a $10 reward. This incentive structure was designed to encourage strategic cost reporting and emulate real-world decision-making scenarios. Participants who failed the attention check were excluded from the remainder of the experiment, and responses with missing data were removed. Ultimately, 396 participants remained in the final sample.

5.2.2. Design and Procedure. At the beginning of the experiment, participants were randomly assigned to one of six groups, each representing a unique combination of allocation mechanism and the principal's budget level. Specifically, we implemented a 2×3 experimental design that varied: (a) the principal's allocation mechanism (our optimal mechanism versus two benchmark mechanisms), and (b) the principal's budget level (low, medium, or high). To reflect the setting in which each agent's cost is private information, each participant was randomly assigned a value drawn from a uniform distribution with support $[c_L, c_H]$.

Participants assigned to the benchmark mechanism groups interacted with both the threshold mechanism and the FPA-based mechanism, presented in randomized order. The parameter choices in the experiment were as follows: the number of agents $N = 3$, minimum testing cost $c_L = 100$, maximal testing cost $c_H = 200$, and regulator's expected payoff $R = 220$. We referred to the principal's budget levels as "low," "middle," and "high" for the corresponding values of 60, 200, and 500, respectively.

Next, the experimental interface guided subjects through the background and purpose of our study. First, participants were informed that they would represent an agent company engaged in R&D of anti-obesity medicine. During the financing process, they would interact with a government funder (the principal) and strategically disclose their private R&D costs to maximize profits. This was because the principal allocated funding based on the reported costs of each agent and those of two competitors. Second, we explained that the purpose of the study was to understand how agents disclose their private cost information in R&D of anti-obesity medicine when interacting with a governmental principal. These insights would contribute to the fields of operations management (OM) and healthcare. Third, we described the payment and incentives. Participants were told: "You will receive \$0.80 for participating in this study. Your goal is to maximize your final return by strategically disclosing your private cost. If your return exceeds that of both competitors, you will receive an additional \$1.20 bonus and be entered into a lottery for a \$10 prize. Please note that only participants who pass the attention check on the next page may proceed and receive the participation fee."

Fourth, participants were introduced to their private cost information. Each agent's R&D cost was drawn from a uniform distribution between 100 and 200 million dollars, which was simplified to a range of 100 to 200 in the experiment. Neither the two competitors nor the principal knows the participant's true cost. The principal may choose to partially fund the agent, with the remaining resources covered by outside sources at the agent's marginal cost, as described in Section 3.

Finally, participants proceeded to the main experiment, having been randomly assigned to one of six mechanism–budget conditions: optimal or benchmark mechanism crossed with low, medium, or high budget levels. Each subject also received a randomly assigned private cost. Participants were informed of the allocation mechanism and the budget condition under which they were operating. They were then presented with theoretical information about the optimal solution for that condition, including the principal's allocation rule and a diagram of the funding allocation curve (Q-curve) under the designed mechanism. Additional details about the experimental instructions and procedure are provided in Appendix EC.2. Participants were reminded that they could report a higher value to pursue greater risk and potential reward, or a lower value to play it safe. Finally, they were asked to input the private cost value they wished to report.

6. Experimental Results

In this section, we analyze the experimental results and test two hypotheses by examining the (absolute) deviations between reported and true private costs, as well as the principal's expected payoff, in Sections 6.1 and 6.2. In addition, we discuss behavioral bias through empirical analysis in Section 6.3.

6.1. Deviations of Reported Cost from True Private Cost

As discussed in Hypothesis 1.1, we expect no significant deviations between agents' reported costs and their true costs under our optimal mechanism. To test this, we conduct paired t-tests to assess whether agents' reported costs differ significantly from their true costs.

Table 1 Statistical Comparison Between Reported Cost and True Private Cost

	Reported Cost ($M_{\text{reported-cost}}$)	True Cost ($M_{\text{true-cost}}$)	Difference	Number of Agents (N)
Low Budget ($B = 60$)	152.258	151.970	0.288 (2.384)	132
Middle Budget ($B = 200$)	148.818	149.197	-0.379 (3.890)	132
High Budget ($B = 500$)	147.758	147.682	0.076 (3.479)	132

Note: (1) *p<0.1; **p<0.05; ***p<0.01; (2) The corresponding p-values for the paired t-tests are 0.33, 0.43, and 0.86 for the low-, middle-, and high-budget conditions, respectively, indicating that we cannot reject the null hypothesis under any budget condition at the 10% significance level.

Table 1 presents the average reported and true costs under different budget conditions, along with the corresponding paired differences and statistical test results. We find that under the low-budget condition, there is no significant difference between reported and true costs ($M_{\text{reported-cost}} = 152.258$, $M_{\text{true-cost}} = 151.970$; $p = 0.33$). Under the medium-budget condition, there is also no significant difference in cost values ($M_{\text{reported-cost}} = 148.818$, $M_{\text{true-cost}} = 149.197$; $p = 0.43$). Similarly, under the high-budget condition, the difference is not significant ($M_{\text{reported-cost}} = 147.758$, $M_{\text{true-cost}} = 147.682$; $p = 0.86$). Based on the above analysis, we find that under all three budget conditions, the reported costs and true private costs are not significantly different at 10% significance level. Hence, the experimental results support Hypothesis 1.1.

In Hypothesis 1.2, we expect that compared to the other two benchmark mechanisms (i.e., the threshold mechanism and the FPA-based mechanism), our optimal mechanism leads to smaller deviations between agents' reported costs and true costs. To test it, we also conduct paired t-tests to evaluate the difference.

Table 2 presents the regression results using the deviations between reported and true costs as the dependent variable, with a treatment dummy for the optimal mechanism. In Column (1) of Panel

A, we find that the optimal mechanism treatment (*Optimal M. Treatment*) leads to significantly lower deviations between reported and true costs compared to the threshold mechanism under the low-budget condition ($\beta = -30.4848,\ p < 0.01$). Columns (2) and (3) yield similar results, showing that the optimal mechanism significantly reduces the deviations under both the medium-budget condition ($\beta = -27.682,\ p < 0.01$) and high-budget condition ($\beta = -24.182,\ p < 0.01$).

Panel B examines whether the optimal mechanism results in smaller deviations compared to the FPA-based mechanism. Across Columns (1) to (3), the estimated coefficients on *Optimal M. Treatment* are consistently negative and statistically significant, indicating that the optimal mechanism significantly reduces the deviations in reported costs relative to the FPA-based benchmark (low-budget: $\beta = -11.212,\ p < 0.01$; medium-budget: $\beta = -9.758,\ p < 0.01$; high-budget: $\beta = -10.970,\ p < 0.01$).

To more intuitively illustrate the deviations of reported costs from true costs across budget levels, we also report the average deviations along with standard error bars. Figure 3 shows that under the optimal mechanism, deviations between reported and true costs are substantially lower than those under the two benchmark mechanisms, thereby supporting Hypothesis 1.2.

One potential concern is that the observed reduction in deviations between reported and true costs under the optimal mechanism may be driven by changes along the intensive margin, that is, agents over-reporting by a smaller amount when they misreport. In that case, the decrease in deviation would not reflect changes along the extensive margin, namely, fewer agents choosing to over-report their costs (i.e., a higher likelihood of truthful reporting). To address this concern, we re-estimate the regression by replacing the dependent variable with the likelihood of over-reporting costs, while keeping the treatment variable (*Optimal M. Treatment*) unchanged. Table A1 in the appendix presents the estimation results. Across Columns (1) to (3) in Panel A, we find that the coefficients on *Optimal M. Treatment* are all negative and statistically significant. The results show that, compared to the threshold mechanism, the optimal mechanism reduces the likelihood of over-reporting costs by more than 60% across all budget conditions. Similarly, Panel B shows that the optimal mechanism decreases the likelihood of over-reporting by at least 63% compared to the FPA-based mechanism. Figure A4 in the appendix further illustrates the average likelihood of overly reporting costs (with standard error bars) for the three budget levels, comparing our optimal mechanism to the two benchmark mechanisms. Therefore, in light of agents' tendency to over-report costs, the empirical evidence continues to support Hypothesis 1.2.

6.2. Principal's Expected Payoff

To test Hypothesis 2, we first calculate the principal's expected payoff from each agent based on Equation (2).[7] Table 3 reports the estimation results using the principal's expected payoff as

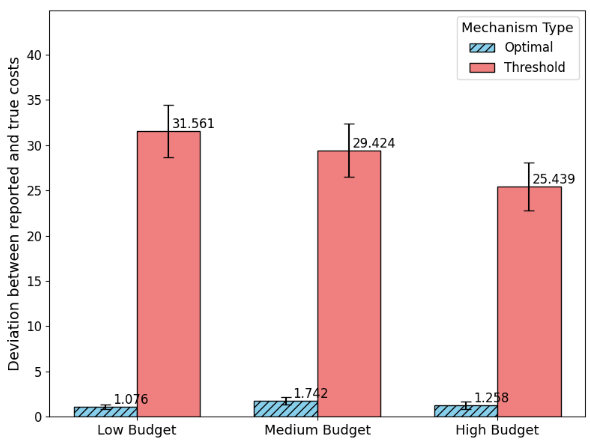
[7] The principal's expected payoff from each agent is U/N, where U is defined by Equation (2).

Table 2 **Deviations between Reported and True Costs – Optimal vs. Benchmark Mechanism Treatments**

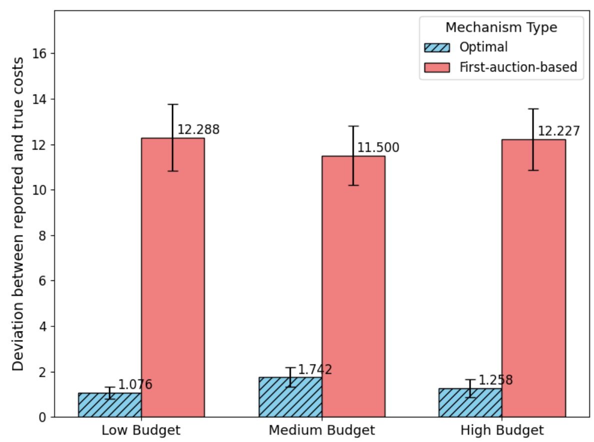
	(1)	(2)	(3)
	Deviations between Reported and True Costs		
	Low Budget	Medium Budget	High Budget
Panel A: Optimal vs. Threshold Mechanism Treatments			
Optimal M. Treatment	-30.485***	-27.682***	-24.182***
	(2.892)	(2.986)	(2.661)
Constant	31.561***	29.424***	25.439***
	(2.880)	(2.955)	(2.631)
Observations	132	132	132
No. of Subjects	132	132	132
Panel B: Optimal vs. FPA-Based Mechanism Treatments			
Optimal M. Treatment	-11.212***	-9.758***	-10.970***
	(1.497)	(1.377)	(1.409)
Constant	12.288***	11.500***	12.227***
	(1.474)	(1.308)	(1.351)
Observations	132	132	132
No. of Subjects	132	132	132

Note: *** p<0.01, ** p<0.05, * p<0.1. The dependent variable is the deviation between reported and true costs.

Figure 3 **Deviations from True Cost: Optimal vs.Threshold and FPA-Based Mechanism Treatments**



(a) Optimal vs. Threshold Mechanism



(b) Optimal vs. FPA-Based Mechanism

the dependent variable and the *Optimal M. Treatment* as the independent variable. In Column (2) of Panel A, we observe that the principal's expected payoff is significantly higher under the optimal mechanism compared to the threshold mechanism ($\beta = -18.884$, $p < 0.05$), suggesting that under the medium-budget condition, our optimal mechanism yields a higher expected payoff for the principal compared to the threshold mechanism. However, in Columns (1) and (3) of Panel A, although the estimated coefficients on *Optimal M. Treatment* are positive, they are not statistically significant, indicating that the optimal mechanism does not significantly increase the principal's expected payoff compared to the threshold mechanism under the low- and high-budget conditions.

Table 3 **Principal's Expected Payoff – Optimal vs. Benchmark Mechanism Treatments**

	(1)	(2)	(3)
	Principal's Expected Payoff		
	Low Budget	Medium Budget	High Budget
Panel A: Optimal vs. Threshold Mechanism Treatments			
Optimal M. Treatment	3.185	18.884**	0.605
	(2.536)	(8.024)	(7.998)
Constant	5.157***	16.948***	36.829***
	(0.347)	(1.249)	(2.086)
Observations	132	132	132
No. of Subjects	132	132	132
Panel B: Optimal vs. FPA-Based Mechanism Treatments			
Optimal M. Treatment	-5.672	-1.368	-17.514**
	(3.617)	(9.304)	(8.401)
Constant	14.014***	37.200***	54.947***
	(2.601)	(4.872)	(3.312)
Observations	132	132	132
No. of Subjects	132	132	132

Note: *** p<0.01, ** p<0.05, * p<0.1. The dependent variable is the principal's expected payoff. Each column represents a budget condition.

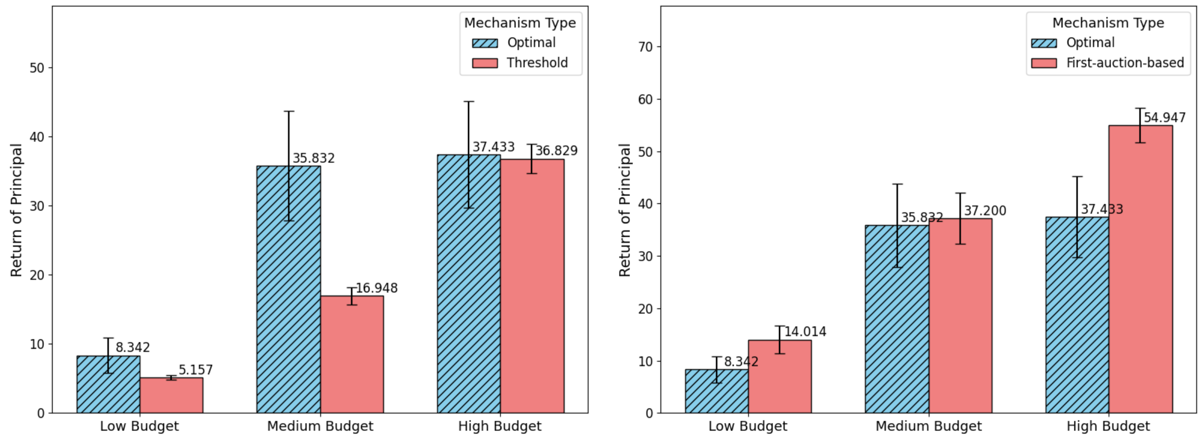
In Panel B of Table 3, we compare the principal's expected payoff between the optimal and the FPA-based mechanism treatments. In Columns (1) and (2), the estimated coefficients on *Optimal M. Treatment* are not statistically significant, suggesting no significant difference between the two mechanisms. Column (3) shows that the optimal mechanism treatment leads to a significantly lower principal's expected payoff relative to the FPA-based mechanism under the high-budget condition ($\beta = -17.514,\ p < 0.05$). Figure 4 displays the principal's average expected payoff under the optimal, threshold, and FPA-based mechanisms. These results show that while the optimal mechanism generally yields higher returns than the threshold mechanism, it leads to lower returns compared to the FPA-based mechanism, thereby failing to support Hypothesis 2.

Since the three agents in our experimental setting compete within the same group, we also compute the principal's expected payoff at the group level. Figure A5 in the appendix further displays the average group-level principal's expected payoff for the optimal, threshold, and FPA-based mechanisms. The results are consistent with the above agent-level analysis.

6.3. Discussion of Behavioral Bias

As shown by the experimental results in Table 3, the theoretically optimal mechanism performs statistically worse than the FPA-based mechanism under high budgets and performs similarly to

Figure 4 Principal's Expected Payoff: Optimal vs. Two Benchmark Mechanism Treatments



(a) Optimal vs. Threshold Mechanism (b) Optimal vs. FPA-Based Mechanism

the FPA-based mechanism under low and medium budgets. These findings contradict **Hypothesis 2**. We therefore discuss potential behavioral explanations for these findings.

Section 6.1 shows that deviations between reported and true costs are not statistically significant, indicating no obvious reporting bias under the optimal mechanism. Therefore, the lack of support for Hypothesis 2 appears to stem from behavioral biases associated with the FPA-based mechanism, which is the main focus of this subsection.

In our experimental setting, *risk aversion* and *ambiguity aversion* are two leading explanations for observed deviations from the risk-neutral bidding benchmark (Pratt 1978, Cox et al. 1988, Salo and Weber 1995). Risk aversion (in our first-price auction environment) means that, holding the underlying environment fixed, bidders place extra weight on securing a safer, more certain payoff and are therefore willing to sacrifice some profit margin in order to increase their chance of winning (Pratt 1978). Ambiguity aversion is the tendency to prefer situations in which the environment and opponents' behavior are clearly and transparently specified over situations in which the relevant features of the environment, or the strategies and valuations of other participants, are uncertain or ill-defined (Ellsberg 1961).

The key distinction is whether the underlying environment is viewed as fixed and well specified (risk) or itself uncertain and ill defined (ambiguity). In auction environments, these forces show up when (i) bidders, facing a given environment, tilt toward more certain payoffs by trading profit margin for a higher chance of winning (risk aversion) and (ii) bidders are unsure about rivals' valuations, strategies, or participation, so the relevant outcome chances are themselves uncertain (ambiguity). Both can induce precautionary bidding relative to the risk-neutral benchmark: in standard models of first-price auctions, risk aversion shifts bids toward safer outcomes (Maskin and Riley 1984, Cox et al. 1988), while under ambiguity, bidders tend to choose more cautious bids, producing systematic departures from the benchmark (Salo and Weber 1995). In our study, the ambiguity-aversion explanation suggests that agents in the FPA-based mechanism, facing uncertainty about the reported costs of their competitors, may adopt conservative reporting strategies. The risk-aversion explanation implies that, even when the underlying environment is held fixed, agents may be willing to sacrifice some expected profit in order to increase their chance of receiving funding, adjusting their reports to secure a safer ranking position.

To identify the two underlying behavioral biases that lead participants to underreport their private costs in our FPA-based mechanism, we consider two experimental approaches: priming and structural manipulations. Priming keeps the economic environment fixed while using short, non-deceptive prompts (for example, a brief text or illustrative example) to make particular considerations, such as payoff risk or uncertainty about others' behavior, more salient. These prompts do not alter payoffs, rules, or opponents; they merely shift participants' attention when choosing bids. Structural manipulations, in contrast, modify the environment itself, for example, by replacing human opponents with robots that follow a fixed strategy, or by switching from stochastic to deterministic payments to eliminate risk.

In our context, full structural control is neither feasible nor fully desirable. Ensuring identical environments across groups would require fixing competitors' strategies and fully specifying the game in each condition, thereby removing the human–human strategic interaction central to our research question. Even then, subjects might still perceive probabilities differently across treatments. Priming avoids these issues. All participants face the same mechanism, information structure, and pool of human opponents; only the salience of payoff risk or strategic uncertainty varies through brief, non-deceptive prompts. This approach offers several practical and methodological advantages. It preserves task comparability because everyone faces the same environment and payoff structure, so differences in bids are more naturally interpreted as differences in attitudes or attention rather than as responses to a different game. It keeps the design simpler and less demanding in terms of sample size, since we do not need additional treatment cells with different payment rules or information regimes. Finally, it better reflects realistic policy interventions, where reframing information is typically more practical than redesigning the allocation mechanism itself.

Therefore, we conducted a second laboratory experiment using priming to identify the two behavioral biases that lead to underreporting under the FPA-based mechanism. In this lab experiment, we recruited 284 participants on MTurk to simulate agents deciding how to report their private costs under the FPA-based mechanism.[8] The experiment was conducted under a high-budget condition, in which the optimal mechanism performs statistically worse than the FPA-based mechanism.

[8] A total of 284 participants were randomly assigned to four groups, with 71 participants in each group. After applying the attention check and excluding individuals who did not complete the experiment, we obtained 266 valid participants. The final group sizes were 69, 64, 67, and 66 participants, respectively.

We adopted a 2×2 experimental design that exogenously manipulated the salience of risk aversion (high vs. low) and ambiguity aversion (high vs. low), creating four experimental groups representing all combinations of the two factors. The logic of the experiment is as follows: if, after exogenously increasing the salience of risk aversion or ambiguity aversion, participants in the treatment groups report significantly lower private costs than those in the corresponding control groups (where salience was not enhanced), this would indicate that risk aversion or ambiguity aversion is an underlying mechanism that leads participants to report their private costs more conservatively. The experimental procedure and rules were similar to those described in Section 5.2.2. Participants were introduced to the experimental background, rules, and incentive structure, which were the same as in the first experiment. They then read the group-specific information (which primed the relevant attitude), received their randomly assigned private cost, and made their reporting decisions.

Specifically, the risk-aversion nudge used to enhance salience stated: "Choosing to report a conservative (lower) cost may increase your chances of receiving funding, but could reduce your eventual profit if funded. Conversely, reporting a higher cost may lower your chance of being funded but increases the likelihood of achieving a higher profit if you are funded. Please consider these trade-offs when deciding what to report." The ambiguity-aversion nudge stated: "Your two competitors' private costs are randomly distributed between 100 and 200, and their reporting strategies are unknown to you. You should consider that the exact reports and strategies of your competitors are unpredictable when choosing what to report." Specifically, under otherwise identical experimental conditions, the first group received a high-risk-aversion information nudge; the second group received a high-ambiguity-aversion information nudge; the third group received both nudges; and the fourth group served as the control group without any nudge. The first information nudge emphasizes that reporting a higher private cost may reduce the likelihood of receiving funding, thereby inducing a high-risk-aversion treatment. The second information nudge highlights the high uncertainty surrounding competitors' private costs and the unpredictability of their reporting strategies, thereby inducing a high-ambiguity-aversion treatment.

After running the lab experiment on MTurk, we built a regression model to examine the above mechanisms empirically.

$$\text{ReportedCost}_i = \alpha + \beta_R \cdot \text{HighRisk}_i + \beta_A \cdot \text{HighAmb}_i + \beta_{RA} \cdot \text{Both}_i + \gamma \cdot \text{Control}_i + \varepsilon_i, \tag{28}$$

where the dependent variable ReportedCost_i represents the strategically reported cost by agent i, and the independent dummy variables HighRisk_i, HighAmb_i, and Both_i equal to 1 if the agent received nudges emphasizing high risk aversion only, high ambiguity aversion only, and both, respectively. Since an agent's true private cost directly influences the strategically reported cost, we include it as a control variable in the regression model.

The regression results are presented in Table 4. We find that all estimated coefficients on $\mathrm{HighRisk}_i$, $\mathrm{HighAmb}_i$, and Both_i are negative and statistically significant at the 1% level. This result confirms that both risk aversion and ambiguity aversion lead agents to report their private costs more conservatively under the FPA-based mechanism in our experimental setting.

Specifically, the estimated coefficient on $\mathrm{HighRisk}_i$ is -7.631, suggesting that exposure to the high-risk-aversion nudge leads agents to underreport their costs by an average of 7.631 points relative to the benchmark group. The estimated coefficient on $\mathrm{HighAmb}_i$ is -7.852, indicating that the high-ambiguity-aversion nudge results in a slightly greater degree of underreporting. The combined informational nudges produce the largest effect, with an estimated coefficient of –8.830, implying the strongest conservative reporting behavior. Finally, consistent with our expectation, the estimated coefficient on the control variable (true private cost) is 0.919 and statistically significant. Taken together, this supplementary lab experiment provides empirical evidence that agents facing the FPA-based mechanism in our setting exhibit behavioral biases, specifically risk aversion and ambiguity aversion, that lead them to report their private costs irrationally and conservatively. These behavioral tendencies explain why the optimal mechanism cannot outperform the FPA-based mechanism in the principal's payoff, particularly under the high budget condition.

Table 4 Examining Risk Aversion and Ambiguity Aversion under FPA-Based Mechanism

	(1)
	Reported Cost
HighRisk	-7.631***
	(1.376)
HighAmb	-7.852***
	(1.403)
Both	-8.830***
	(1.393)
Control	0.919***
	(0.0173)
Constant	30.08***
	(2.824)
Observations	266
R-squared	0.919

Note: *** p<0.01, ** p<0.05, * p<0.1. The control variable is the agent's true private cost.

7. Conclusions

This paper addresses the funding allocation challenge that governments face in the R&D of anti-obesity medicine and proposes an effective approach supported by both theoretical and experimental analysis. Our mechanism enables governments to elicit truthful private information from pharmaceutical firms when investing in anti-obesity medicine R&D, which is essential for advancing broader public health objectives. Theoretically, our mechanism incentivizes pharmaceutical firms to truthfully report their private R&D costs while maximizing the government's expected payoff. However, practical outcomes may diverge from these theoretical predictions due to violations of the rationality assumption and the presence of behavioral biases such as risk aversion and ambiguity aversion. Therefore, we design laboratory experiments to evaluate the performance of the theoretically optimal mechanism in practice and to investigate behavioral explanations for discrepancies between theory and empirical performance. In our study, we find that the empirical results of the optimal mechanism closely align with theoretical expectations. Although the performance of the first-price auction-based mechanism differs dramatically between theory and experiment due to risk and ambiguity aversion, our optimal mechanism remains effective in inducing truthful cost reporting and sustaining government payoffs as much as possible.

Based on these findings, our mechanism offers a promising tool for governments, other public sectors, and health-related NGOs to gather reliable market information in drug R&D settings. This is particularly suitable for treatments such as anti-obesity medicine, where firms hold a significant informational advantage regarding development costs. Therefore, this mechanism can inform subsidy design, supply curve estimation, and long-term public health planning related to anti-obesity medicine R&D. Furthermore, our findings highlight the importance of accounting for behavioral biases when designing drug R&D investment mechanisms. Governments should consider experimental validation, such as through laboratory experiments, before implementing the theoretically optimal mechanisms in practice. Such efforts are crucial for evaluating the real-world applicability of theoretical findings. Overall, our paper offers insights to help governments design incentive mechanisms that elicit truthful information from firms engaged in anti-obesity medicine R&D, thereby supporting effective treatment development and reducing the broader societal burden of obesity.

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Online Appendix

EC.1. Parameter Calculations in the Experimental Setting

We set $N = 3$, $c_L = 100$, $c_H = 200$, $R = 220$, and f is the density of a uniform distribution on $[c_L, c_H]$, i.e., $F(c) = (c - c_L)/(c_H - c_L)$ for any $c \in [c_L, c_H]$.

EC.1.1. Optimal Mechanism

Case 1 (low budget: $B = 60$). In this case, we have $\tilde{c} \approx 148.32$, and the corresponding allocation rule is given by

$$q(c_1, c_{-1}) = Q(c_1) = \begin{cases} 0.012\, c - 2.4\, \log(c) + 10.33, & \text{if } c_1 \le \tilde{c}, \\ 0, & \text{otherwise,} \end{cases} \tag{EC.1}$$

and

$$m(c_1, c_{-1}) = \begin{cases} B, & \text{if } c_1 < \min_{j \in \{2,\dots,N\}} c_j \text{ and } c_1 \le \tilde{c}, \\ 0, & \text{otherwise.} \end{cases} \tag{EC.2}$$

Let c_1 denote firm 1's reported type. The mechanism is implemented as follows:

- If $c_1 \le \bar{c}$, the principal invests in firm 1's project and claims a share $Q(c_1)$ of the project's return. Additionally, if firm 1 reports the lowest type among all participating agents, it receives a payment of B from the principal.
- If $c_1 > \bar{c}$, the principal does not claim firm 1's project, and firm 1 receives no compensation.

In this scenario, the principal's optimal expected payoff is given by

$$\frac{3}{100} \int_{100}^{148.32} \left[220\, [0.012\, c - 2.4\, \log(c) + 10.33] - 60 \left(\frac{200 - c}{100} \right)^2 \right] \mathrm{d}c \approx 32.31.$$

Case 2 (medium budget: $B = 200$). In this case, we have $\bar{c} \approx 132.7$ and $\check{c} \approx 153$, and the corresponding allocation rule is given by

$$q(c_1, c_{-1}) = Q(c_1) = \begin{cases} 1, & \forall c_1 \in [c_L, \bar{c}], \\ 0.04\, c - 8\, \log(c) + 34.41, & \forall c_1 \in (\bar{c}, \check{c}], \\ 0, & \forall c_1 \in (\check{c}, c_H], \end{cases}$$

and

$$m(c_1, c_{-1}) = \begin{cases} B/|S|, & \text{if } c_1 \le \bar{c}, \\ B, & \text{if } c_1 < \min_{j \in \{2,\dots,N\}} c_j \text{ and } \bar{c} < c_1 \le \check{c}, \\ 0, & \text{otherwise,} \end{cases}$$

where $S \equiv \{k \in [N] : c_k \le \bar{c}\}$.

Let c_1 denote firm 1's reported type. The mechanism is implemented as follows:

- If $c_1 \le \check{c}$, the principal claims a share $Q(c_1)$ of the project;
- If $c_1 \le \bar{c}$, firm 1 receives a transfer of B/S from the principal, where S denotes the number of firms whose reported types are below $\bar{c}$;
- If $\bar{c} < c_1 \le \check{c}$, firm 1 receives a transfer of B from the principal only if its reported cost is the lowest among all N agents;

- If $c_1 > \check{c}$, the principal does not claim firm 1's project, and firm 1 receives no payment.

In this scenario, the principal's optimal expected payoff is given by

$$\frac{3}{100}\left\{\int_{100}^{132.7}\left[220-132.7\left[\int_{132.7}^{153}\frac{1}{y^2}\left(\frac{200-y}{100}\right)^2\mathrm{d}y+1\right]\right]\mathrm{d}c\right.$$
$$\left.+\int_{132.7}^{153}\left[220\left[0.04\,c-8\,\log(c)+34.41\right]-200\left(\frac{200-c}{100}\right)^2\right]\mathrm{d}c\right\}\approx 103.86.$$

Case 3 (high budget: $B=500$). In this case, we have $\check{c}=160$, and the corresponding allocation rule is given by

$$q(c_1,c_{-1})=Q(c_1)=\begin{cases}1, \ \forall c_1\in[c_L,\check{c}],\\ 0, \ \forall c_1\in(\check{c},c_H],\end{cases}$$

and

$$m(c_1,c_{-1})=\begin{cases}307.7/|S|, & \text{if } c_1\le\check{c},\\ 0, & \text{otherwise},\end{cases}$$

where $S\equiv\{k\in[N]:c_k\le\check{c}\}$.

Let c_1 denote firm 1's reported type. The mechanism is implemented as follows:

- If $c_1\le\check{c}$, the principal claims the entire project and provides a transfer of $\bar{B}/S$ to firm 1, where $\bar{B}$ is a constant defined in Proposition 4 and S denotes the number of firms whose reported types are below $\check{c}$;
- If $c_1>\check{c}$, the principal does not claim firm 1's project, and firm 1 receives no compensation.

In this scenario, the principal's optimal expected payoff is given by

$$\frac{3}{100}\int_{100}^{160}(220-160)\,\mathrm{d}c=108.$$

EC.1.2. Threshold Mechanism

In this setting, the principal allocates the resource according to a simple threshold mechanism. We fix the threshold at 175.

The rule is as follows: when firm 1's reported cost exceeds the threshold (175), the principal will not claim its project, and the corresponding monetary transfer is 0. Otherwise, firm 1 will share the budget equally with all its competitors whose reports are below 175. The corresponding allocation and payment rules are given by

$$q(c_1,c_{-1})=\begin{cases}\min\left\{\frac{B}{175\,S},1\right\}, & \text{if } c_1\le 175,\\ 0, & \text{if } c_1>175,\end{cases}\qquad m(c_1,c_{-1})=175\,q(c_1,c_{-1}),$$

where S is the number of firms reporting below 175. Note that the maximum compensation the firm can receive is 175.

For instance, suppose $B=200$, and three agents report their costs as 110, 130, and 150, respectively. In this case, the funder claims a share of $200/(3\cdot 175)$ from each of the three firms, and each firm receives a funding of $200/3$.

EC.1.3. FPA-Based Mechanism

In this setting, the principal adopts a first-price auction-style rule to allocate the limited resources.

Let c_i^* denote the i-th lowest report from the firm. After each firm makes its report, the principal allocates as follows:

- The firm reporting the lowest cost (denoted by c_1^*) will be claimed a share of $\min\left\{\frac{B}{c_1^*}, 1\right\}$ and receive $c_1^* \cdot \min\left\{\frac{B}{c_1^*}, 1\right\}$.
- If there is additional budget after the first round of allocation, the firm reporting the second lowest cost (denoted by c_2^*) will be claimed a share of $\min\left\{\frac{B-c_1^*}{c_2^*}, 1\right\}$ and receive $c_2^* \cdot \min\left\{\frac{B-c_1^*}{c_2^*}, 1\right\}$.
- The principal repeats this procedure until the budget is exhausted or all the N agents' requests are satisfied.

For instance, suppose $B = 200$, and three agents report their costs as 110, 130, and 150, respectively. In this case, the agent reporting 110 will be claimed the whole project and receive 110, and the agent reporting 130 will be claimed a share $(200 - 110)/130$ of its project and receive the remaining 90. Since the budget is exhausted, the agent reporting 150 will not develop the project for the principal.

EC.2. Detailed Experimental Procedure

EC.2.1. Introduction to the Background of Experiment

We first present the background of the experiment and the purpose of the study to the participants, as follows:

You are representing an agent company engaged in the research and development (R&D) of anti-obesity medicine. During the funding process, you will interact with a government principal (represented by our research team) and must strategically disclose your R&D costs (i.e., private costs) to maximize your profits. This is because the government principal allocates investment funds based on your disclosed costs as well as those disclosed by two competing agents.

The purpose of this study is to understand how agents strategically disclose private costs in the context of R&D for anti-obesity medicine, under different funding allocation mechanisms introduced by the principal. We expect that the findings will contribute to research in operations management and healthcare. Participation in this study is expected to take approximately 15 minutes.

Payment. First, you will receive 80 cents for participating in this study. Your goal is to maximize your final return through strategically disclosing your cost. If your return exceeds that of both your competitors, you will receive an additional \$1.2 reward and be entered into a lottery for a \$10 bonus. Please note that only participants who pass the simple attention check on the following page can proceed with the experiment and receive the participation fee.

Cost-related Information. (1) Each simulated agent's R&D cost is uniformly distributed between 100 million and 200 million, which is simplified as 100–200 in the experiment. (2) You will receive your own private R&D cost at the beginning of the study. (3) Neither your two competitors nor the principal (represented by the researchers) will know your true cost during the experiment. (4) Even if the principal's total budget is lower than your true cost, it doesn't matter — the investor only claims part of the project, and you will receive the full amount of funding from other sources, meaning Obesity Medicine will still be successfully developed.

Informed Consent. By clicking "yes" below, you are indicating that you consent to participate in this research study.

EC.2.2. Attention Check

We then administer an attention check using a single-choice question, asking participants to select the largest number among four randomly ordered numerical options. This attention check helps us screen out participants who are not reading carefully and are merely attempting to complete the study for compensation.

EC.2.3. Main Instruction

Each subject was randomly assigned to one of six groups: the optimal mechanism–low budget group, optimal mechanism–medium budget group, optimal mechanism–high budget group, benchmark mechanism–low budget group, benchmark mechanism–medium budget group, and benchmark mechanism–high budget group.[9] The low, medium, and high budget levels were set at 50, 200, and 500, respectively. In addition, each subject received a randomly assigned private cost, which was an integer between 100 and 200.

The total value generated for the principal from the completion of a project was fixed at 220. Subjects were then informed about the specific allocation mechanism and budget condition they were operating under. They were also presented with theoretical information regarding the optimal strategy in that condition, including the principal's allocation rule and a visual representation of the funding allocation curve under the corresponding mechanism. Details of the principal's allocation rules under the optimal mechanism, the threshold mechanism, and the FPA-based mechanism are provided in Appendix Section EC.1. The funding allocation curve under our optimal mechanism is illustrated in Figure 1.

For example, under the optimal mechanism, agents can understand that truthfully reporting their private cost would yield the highest theoretical profit. In contrast, under the FPA-based mechanism, they recognize the importance of strategically reporting slightly higher costs to increase

[9] When agents were assigned to the benchmark mechanism group, they were presented with the threshold and FPA-based mechanism conditions in a random order, and were asked to input their reported cost under each condition.

their ex post payoffs upon receiving funding, while avoiding excessive over-reporting that could reduce their chances of being funded. Finally, each subject was asked to write down the private cost they chose to report strategically.

EC.3. Additional Tables and Figures

Figure A1 Illustration of the implementation rule defined in Corollary 1 for the case $N = 2$. We use $q_1 \equiv q(c_1, c_2)$ and $m_1 \equiv m(c_1, c_2)$ to denote the allocation and payment associated with firm 1. Since the mechanism is symmetric, the implementation for firm 2 is omitted for simplicity.

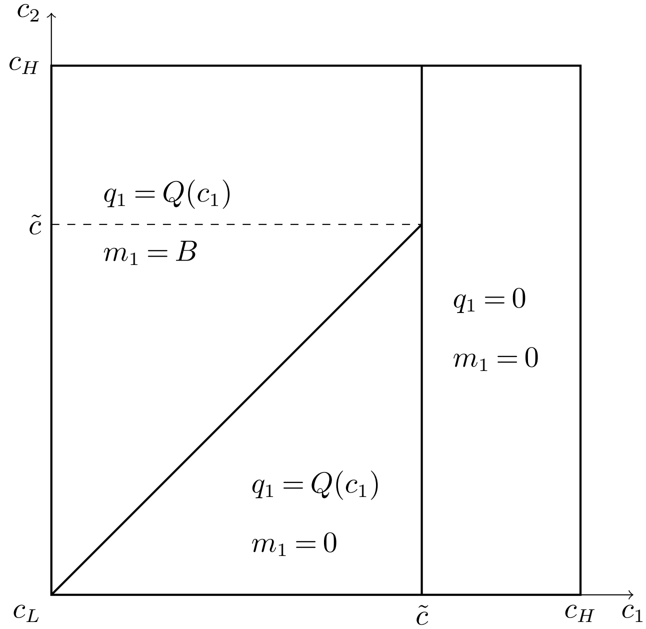


Figure A2 **Illustration of the implementation rule defined in Corollary 3 for the case $N = 2$. We use $q_1 \equiv q(c_1, c_2)$ and $m_1 \equiv m(c_1, c_2)$ to denote the allocation and payment associated with firm 1. Since the mechanism is symmetric, the implementation for firm 2 is omitted for simplicity.**

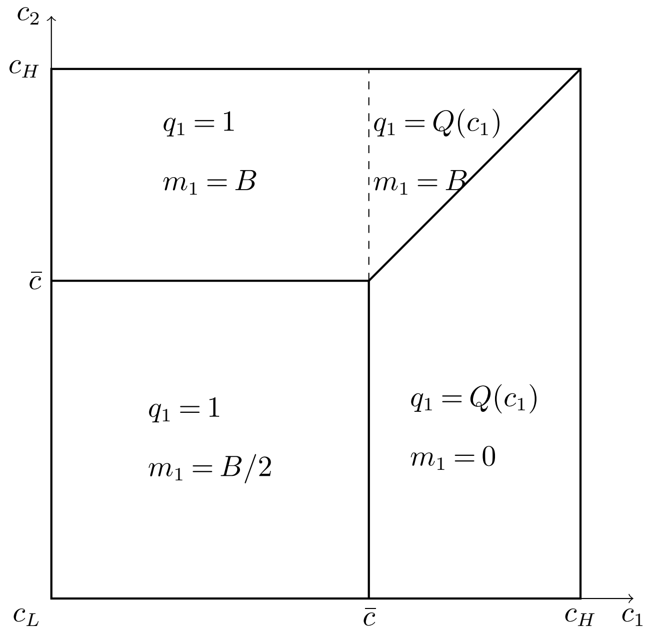


Figure A3 **Illustration of the implementation rule defined in Corollary 4 for the case $N = 2$. We use $q_1 \equiv q(c_1, c_2)$ and $m_1 \equiv m(c_1, c_2)$ to denote the allocation and payment associated with firm 1. Since the mechanism is symmetric, the implementation for firm 2 is omitted for simplicity.**

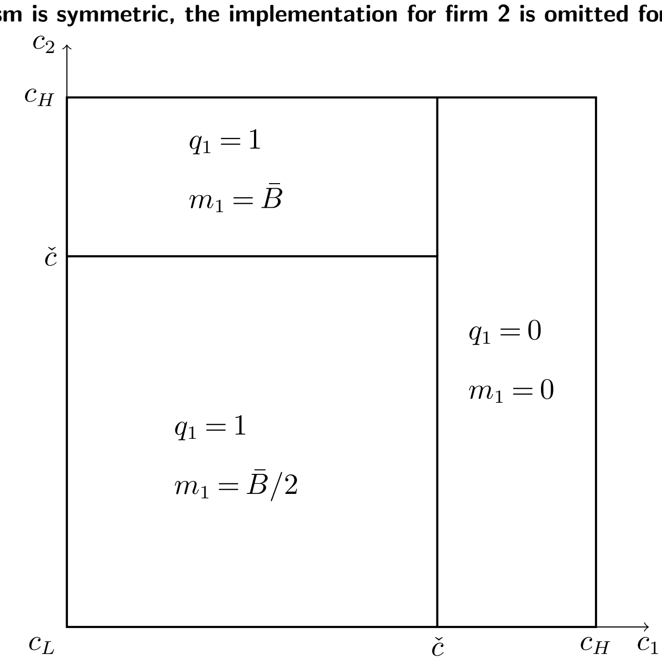
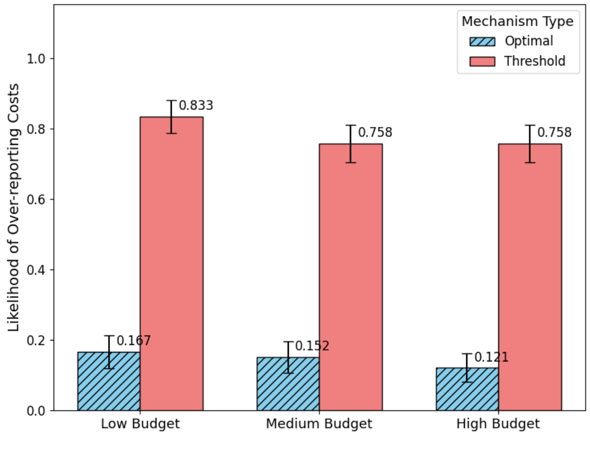


Table A1 Likelihood of Over-reporting Costs – Optimal vs. Benchmark Mechanism Treatments

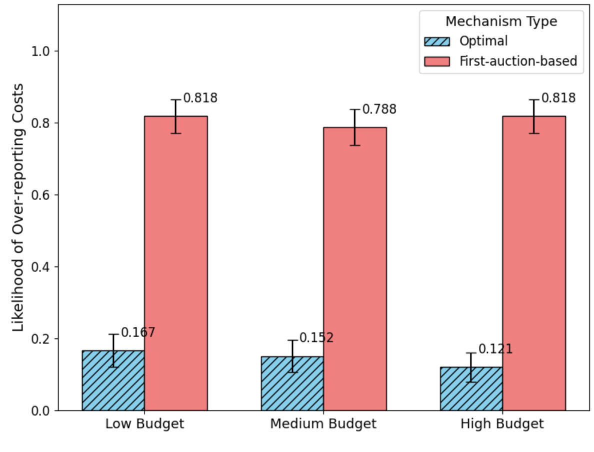
	(1)	(2)	(3)
	Likelihood of Over-reporting Costs		
	Low Budget	Medium Budget	High Budget
Panel A: Optimal vs. Threshold Mechanism Treatments			
Optimal M. Treatment	-0.6667***	-0.6061***	-0.6364***
	(0.065)	(0.069)	(0.067)
Constant	0.8333***	0.7576***	0.7576***
	(0.046)	(0.053)	(0.053)
Observations	132	132	132
No. of Subjects	132	132	132
Panel B: Optimal vs. FPA-Based Mechanism Treatments			
Optimal M. Treatment	-0.6515***	-0.6364***	-0.6970***
	(0.067)	(0.067)	(0.063)
Constant	0.8182***	0.7879***	0.8182***
	(0.048)	(0.051)	(0.048)
Observations	132	132	132
No. of Subjects	132	132	132

Note: *** p<0.01, ** p<0.05, * p<0.1. The dependent variable is the deviation between reported and true costs.

Figure A4 Likelihood of Over-reporting Costs: Optimal vs.Threshold and FPA-Based Mechanism Treatments

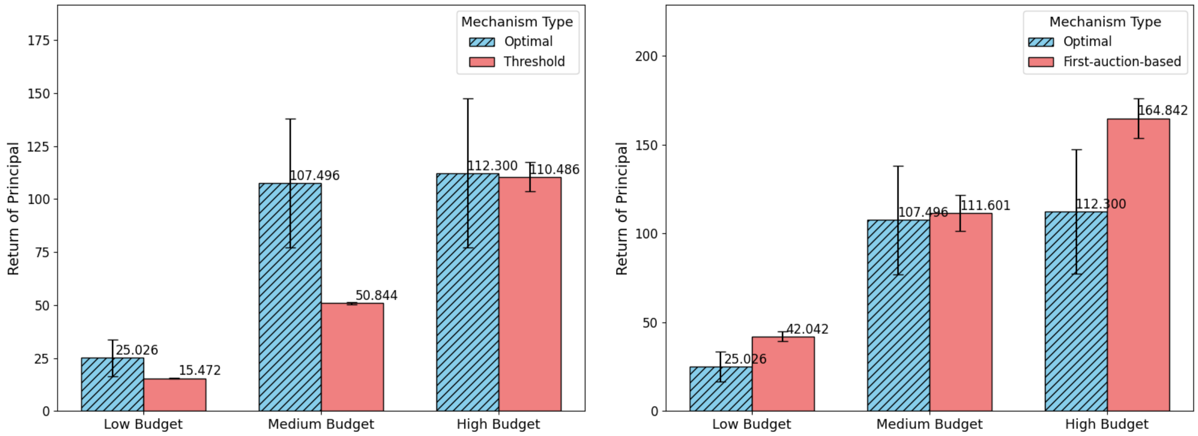


(a) Optimal vs. Threshold Mechanism



(b) Optimal vs. FPA-Based Mechanism

Figure A5 Principal's Expected Payoff (Group Level): Optimal vs. Two Benchmark Mechanism Treatments



(a) Optimal vs. Threshold Mechanism (b) Optimal vs. FPA-Based Mechanism

EC.4. Proofs

The following lemma is standard in mechanism design settings with direct payments between the principal and the agents. For completeness, we state it here.

LEMMA EC.1. *The incentive constraints* (IC) *and* (IR) *hold if and only if*

$$Q \text{ non-increasing}, \tag{EC.3}$$
$$M(c) = u(c_H) + \int_c^{c_H} Q(y)\,\mathrm{d}y + c\,Q(c), \text{ and} \tag{EC.4}$$
$$u(c_H) = 0, \tag{EC.5}$$

where $u(c) \equiv \hat{u}(c,c)$.

Proof of Lemma EC.1 This result is standard. See Proposition 5.2 on page 66 of Krishna (2009). □

PROPOSITION EC.1. *The funder's optimization problem* (12) *can be rewritten as follows:*

$$\max_{Q\in\Omega'} \int_{c_L}^{c_H} w(c)\,Q(c)\,f(c)\,\mathrm{d}c, \tag{EC.6}$$

where the virtual valuation function w *is defined by*

$$w(c) \equiv R - c - \frac{F(c)}{f(c)}, \quad \forall c\in[c_L, c_H],$$

and the feasible set Ω' *is determined by linear constraints* (10),(EC.3)*, and*

$$\int_{c_L}^{c}\left[\int_y^{c_H} Q(z)\,\mathrm{d}z + y\,Q(y)\right]\mathrm{d}F(y) \le \frac{B}{N}\left[1-[1-F(c)]^N\right], \quad \forall c\in[c_L, c_H].$$

Proof of Proposition EC.1 Consider the funder's problem (12). Substituting M in the objective function with (EC.4) yields

$$\begin{aligned}\int_{c_L}^{c_H}[R\,Q(c)-M(c)]\,\mathrm{d}F(c) &= \int_{c_L}^{c_H}\left[R\,Q(c)-\left[\int_c^{c_H}Q(y)\,\mathrm{d}y + c\,Q(c)\right]\right]\mathrm{d}F(c)\\ &= \int_{c_L}^{c_H}\underbrace{\left[R-c-\frac{F(c)}{f(c)}\right]}_{=w(c)}Q(c)\,f(c)\,\mathrm{d}c.\end{aligned}$$

The Border's constraint in (9) follows from substituting the expression for M given in (EC.4). Moreover, the nonnegativity of M is implied by the nonnegativity of Q.

This completes the proof. □

Proof of Proposition 1 First, we verify the feasibility of our candidate solution. For any $c\in[c_L,\tilde{c}]$, differentiating Q yields

$$Q'(c) = -\frac{B\,(N-1)\,[1-F(c)]^{N-2}\,f(c)}{c} < 0.$$

Therefore, the desired feasibility condition boils down to

$$Q(c_L) \le 1 \quad \Leftrightarrow \quad B \le \left[\frac{1}{c_L} - \int_{c_L}^{\tilde{c}} \frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y\right]^{-1}$$

and

$$Q(\tilde{c}) \ge 0 \quad \Leftrightarrow \quad \frac{[1-F(\tilde{c})]^{N-1}}{\tilde{c}} \ge 0.$$

Both conditions hold automatically. Since the Border's constraint (9) is binding on $[c_L, \tilde{c}]$, the desired feasibility condition also holds.

Next, we prove the optimality of our candidate solution. Define

$$\begin{aligned} &\xi(c) = -J'(c), && \forall c \in [c_L, \tilde{c}], \\ &\delta(c) = -w(c)\,f(c) + w(\tilde{c})\,f(\tilde{c}), && \forall c \in [\tilde{c}, c_H], \end{aligned}$$

where

$$J(c) \equiv \frac{N(c)}{c^2\,f(c)} \tag{EC.7}$$

and

$$N(c) \equiv c\,w(c)\,f(c) - \tilde{c}\,w(\tilde{c})\,f(\tilde{c}) + \int_c^{\tilde{c}} w(y)\,f(y)\,\mathrm{d}y. \tag{EC.8}$$

We have

$$N(c) = -\int_c^{\tilde{c}} y\,\mathrm{d}\left(w(y)\,f(y)\right) \ge 0,$$

where the monotonicity of $w(c)\,f(c)$ is guaranteed by Assumption 1, i.e.,

$$\frac{\mathrm{d}}{\mathrm{d}c}\left[w(c)\,f(c)\right] = (R-c)\,f'(c) - 2\,f(c) < 0.$$

The nonnegativity of ξ is equivalent to the monotonicity of

$$\begin{aligned} \frac{1}{c^2\,f(c)}\left[c\,w(c)\,f(c) - \tilde{c}\,w(\tilde{c})\,f(\tilde{c}) + \int_c^{\tilde{c}} w(y)\,f(y)\,\mathrm{d}y\right] &= \frac{1}{c^2\,f(c)}\left[c\,w(c)\,f(c) - \int_{c_L}^{c} w(y)\,f(y)\,\mathrm{d}y\right] \\ &= \frac{1}{c^2\,f(c)}\left[c\,w(c)\,f(c) - (R-c)\,F(c)\right], \end{aligned}$$

We have

$$\begin{aligned} \frac{1}{c^2\,f(c)}\left[c\,w(c)\,f(c) - (R-c)\,F(c)\right] \text{ non-increasing} \quad &\Leftrightarrow \quad 2\,F(c)\,f(c) + c\,F(c)\,f'(c) - 2\,c\,f(c)^2 \le 0, \\ &\Leftarrow \quad 2\,f(c) + c\,f'(c) \le 0. \end{aligned}$$

Moreover, we have

$$\frac{N(c)}{c^2\,f(c)} \text{ non-increasing} \quad \Leftrightarrow \quad c^2\,f(c) \underbrace{N'(c)}_{=c\,\frac{\mathrm{d}}{\mathrm{d}c}(w(c)\,f(c)) \le 0} - c\,N(c)\,\left[2\,f(c) + c\,f'(c)\right] \le 0,$$

which also holds when $2\,f(c)+c\,f'(c)\geq 0$. Therefore, we conclude that ξ is nonnegative on $[c_L,\tilde{c}]$.

The nonnegativity of δ follows from the monotonicity of $w(c)\,f(c)$ directly.

For any $c\in[c_L,\tilde{c}]$, we have

$$\xi(c)=-J'(c)\quad\Rightarrow\quad\int_c^{\tilde{c}}\xi(y)\,dy=J(c),$$

where the last step follows from $J(\tilde{c})=0$. Therefore, we have

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}c}\left[\int_{c_L}^{c}\left(\int_y^{\tilde{c}}\xi(z)\,\mathrm{d}z\right)f(y)\,\mathrm{d}y+c\,f(c)\int_c^{\tilde{c}}\xi(y)\,\mathrm{d}y\right]\\
=\;&[2\,f(c)+c\,f'(c)]\int_c^{\tilde{c}}\xi(y)\,\mathrm{d}y-c\,f(c)\,\xi(c)\\
=\;&[2\,f(c)+c\,f'(c)]\;J(c)-c\,f(c)\,\xi(c)\\
=\;&\frac{-c^2\,w(c)\,f(c)^2+c^3\,w'(c)\,f(c)^2+2\,c\,(R-c)\,F(c)\,f(c)+c^2\,F(c)\,f(c)+c^2(R-c)\,F(c)\,f'(c)-c^2(R-c)\,f(c)^2}{c^3\,f(c)}\\
&+\frac{2\,f(c)+c\,f'(c)}{c^2\,f(c)}\,[c\,w(c)\,f(c)-(R-c)\,F(c)]\\
=\;&(R-c)\,f'(c)-2\,f(c)\\
=\;&\frac{\mathrm{d}}{\mathrm{d}c}\,[w(c)\,f(c)]\,,
\end{aligned}$$

where the fourth equality follows from substituting the definition of w and simplifying the resulting expression. Since

$$\left[\int_{c_L}^{c}\left(\int_y^{\tilde{c}}\xi(z)\,\mathrm{d}z\right)f(y)\,\mathrm{d}y+c\,f(c)\int_c^{\tilde{c}}\xi(y)\,\mathrm{d}y\right]_{c=c_L}=c_L\,f(c_L)\,J(c_L)=w(c_L)\,f(c_L),$$

we know that

$$\int_{c_L}^{c}\left(\int_y^{\tilde{c}}\xi(z)\,\mathrm{d}z\right)f(y)\,\mathrm{d}y+c\,f(c)\int_c^{\tilde{c}}\xi(y)\,\mathrm{d}y=w(c)\,f(c),\quad\forall c\in[c_L,\tilde{c}].$$

For any $c\in[\tilde{c},c_H]$, by definition, we have

$$\int_{c_L}^{\tilde{c}}\left(\int_y^{\tilde{c}}\xi(z)\,\mathrm{d}z\right)f(y)\,\mathrm{d}y-\delta(c)=w(c)\,f(c).$$

Because

$$\begin{aligned}
&\int_{c_L}^{\tilde{c}}\left[\int_{c_L}^{c}\left[\int_y^{c_H}Q(z)\,\mathrm{d}z+y\,Q(y)\right]\mathrm{d}F(y)\right]\xi(c)\,\mathrm{d}c+\int_{\tilde{c}}^{c_H}[-Q(c)]\;\delta(c)\,\mathrm{d}c\\
=\;&\int_{c_L}^{\tilde{c}}\left[\int_{c_L}^{c}\left(\int_y^{\tilde{c}}\xi(z)\,\mathrm{d}z\right)f(y)\,\mathrm{d}y+c\,f(c)\int_c^{\tilde{c}}\xi(y)\,\mathrm{d}y\right]Q(c)\,\mathrm{d}c+\int_{\tilde{c}}^{c_H}\left[\int_{c_L}^{\tilde{c}}\left(\int_y^{\tilde{c}}\xi(z)\,\mathrm{d}z\right)f(y)\,\mathrm{d}y-\delta(c)\right]Q(c)\,\mathrm{d}c\\
=\;&\int_{c_L}^{c_H}w(c)\,Q(c)\,f(c),
\end{aligned}$$

and hence, the objective function is bounded above by

$$\frac{B}{N}\int_{c_L}^{\tilde{c}}\left[1-[1-F(c)]^N\right]\xi(c)\,\mathrm{d}c.$$

Our candidate solution achieves this upper bound, as shown by

$$
\begin{aligned}
& \frac{B}{N} \int_{c_L}^{\tilde{c}} \left[1-[1-F(c)]^N\right] \xi(c)\,\mathrm{d}c - \int_{c_L}^{\tilde{c}} w(c)\,Q(c)\,f(c)\,\mathrm{d}c \\
= & -\frac{B}{N} \int_{c_L}^{\tilde{c}} \left[1-[1-F(c)]^N\right] \mathrm{d}J(c) - B \int_{c_L}^{\tilde{c}} w(c) \left[\frac{[1-F(c)]^{N-1}}{c} - \int_c^{\tilde{c}} \frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y\right] f(c)\,\mathrm{d}c \\
= & -\frac{B}{N} \left[\left[1-[1-F(\tilde{c})]^N\right] J(\tilde{c}) - N \int_c^{\tilde{c}} J(c)\,[1-F(c)]^{N-1}\,f(c)\,\mathrm{d}c\right] \\
& - B \int_{c_L}^{\tilde{c}} w(c) \left[\frac{[1-F(c)]^{N-1}}{c} - \int_c^{\tilde{c}} \frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y\right] f(c)\,\mathrm{d}c \\
= & -B \int_c^{\tilde{c}} \frac{1}{c^2}(R-c)\,F(c)\,[1-F(c)]^{N-1}\,\mathrm{d}c + B \int_{c_L}^{\tilde{c}} w(c) \left[\int_c^{\tilde{c}} \frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y\right] f(c)\,\mathrm{d}c \\
= & -B \int_c^{\tilde{c}} \frac{1}{c^2}(R-c)\,F(c)\,[1-F(c)]^{N-1}\,\mathrm{d}c + B \int_{c_L}^{\tilde{c}} \left(\int_{c_L}^{y} w(c)\,f(c)\,\mathrm{d}c\right) \frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y \\
= & -B \int_c^{\tilde{c}} \frac{1}{c^2}(R-c)\,F(c)\,[1-F(c)]^{N-1}\,\mathrm{d}c + B \int_{c_L}^{\tilde{c}} (R-y)\,F(y)\,\frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y \\
= & \ 0,
\end{aligned}
$$

and thus establishes its optimality.

This completes the proof. ☐

Proof of Corollary 1 By construction, for any $c_1 \in [c_L, c_H]$, we have

$$\int_{c_{-i} \in [c_L, c_H]^{N-1}} q(c_i, c_{-i}) \prod_{k \in [N] \setminus \{i\}} \mathrm{d}F(c_k) = Q(c_i)$$

and

$$\int_{c_{-i} \in [c_L, c_H]^{N-1}} m(c_i, c_{-i}) \prod_{k \in [N] \setminus \{i\}} \mathrm{d}F(c_k) = B\,[1-F(c_i)]^{N-1} = M(c_i).$$

Meanwhile, for any reporting profile $\mathbf{c} \in [c_L, c_H]^N$, we have $\sum_{i=1}^N m(c_i, c_{-i}) \le B$ and $0 \le q(c_i, c_{-i}) \le 1$, implying its feasibility.

This completes the proof. ☐

Proof of Proposition 2 We begin by verifying the feasibility of our candidate solution, starting with the existence of the thresholds $\bar{c}$ and $\check{c}$. Define

$$h(c) = -c\,w(c)\,f(c) + \int_{c_L}^{c} w(y)\,f(y)\,\mathrm{d}y, \quad \forall c \in [c_L, c_H]. \tag{EC.9}$$

The first equation in (20), which characterizes the relationship between $\bar{c}$ and $\check{c}$, can be rewritten as

$$[F(\bar{c}) + \bar{c}\,f(\bar{c})]\,[h(\check{c}) - h(\bar{c})] = \bar{c}^2\,w(\bar{c})\,f(\bar{c})^2 \quad \Rightarrow \quad h(\check{c}) = h(\bar{c}) + \frac{\bar{c}^2\,w(\bar{c})\,f(\bar{c})^2}{F(\bar{c}) + \bar{c}\,f(\bar{c})}.$$

Since h is monotonic, for any fixed $\bar{c}$, there exists a unique $\check{c}$ satisfying this equation. Let $g(\bar{c})$ denote this solution. That is,

$$h(g(\bar{c})) = h(\bar{c}) + \frac{\bar{c}^2\, w(\bar{c})\, f(\bar{c})^2}{F(\bar{c}) + \bar{c}\, f(\bar{c})} \quad \Rightarrow \quad g(\bar{c}) = h^{-1}\left(h(\bar{c}) + \frac{\bar{c}^2\, w(\bar{c})\, f(\bar{c})^2}{F(\bar{c}) + \bar{c}\, f(\bar{c})}\right).$$

We then prove the monotonicity of function g. We have

$$\begin{aligned} h(c) + \frac{c^2\, w(c)\, f(c)^2}{F(c) + c\, f(c)} &= -c\, w(c)\, f(c) + (R - c)\, F(c) + \frac{c^2\, w(c)\, f(c)^2}{F(c) + c\, f(c)} \\ &= R\, \underbrace{\left[F(c) - c\, f(c) + \frac{c^2 f(c)^2}{c\, f(c) + F(c)}\right]}_{\equiv \Upsilon(c)}. \end{aligned}$$

Differentiating Υ yields

$$\Upsilon'(c) = -c\, f'(c) + \frac{[2\, c\, f(c)^2 + 2\, c^2\, f(c)\, f'(c)]\ [c\, f(c) + F(c)] - c^2\, f(c)^2\ [2\, f(c) + c\, f'(c)]}{[c\, f(c) + F(c)]^2} > 0,$$

where the inequality follows from

$$\begin{aligned} & -c\, f'(c)\, [c\, f(c) + F(c)]^2 + \left[2\, c\, f(c)^2 + 2\, c^2\, f(c)\, f'(c)\right]\ [c\, f(c) + F(c)] - c^2\, f(c)^2\ [2\, f(c) + c\, f'(c)] \\ = {} & c\, F(c)\ \left[-F(c)\, f'(c) + 2\, f(c)^2\right] \\ > {} & 0. \end{aligned}$$

Combining the monotonicity of the function $h(c) + \frac{c^2\, w(c)\, f(c)^2}{F(c) + c\, f(c)}$ with that of h yields the desired result.

Define

$$B(\bar{c}) = \frac{\bar{c}\, F(\bar{c})}{\frac{1}{N}[1 - [1 - F(\bar{c})]^N] - \bar{c}\, F(\bar{c})\ \int_{\bar{c}}^{g(\bar{c})} \frac{[1 - F(y)]^{N-1}}{y^2}\, \mathrm{d}y},$$

which represents the unique budget level for which the second equation in (20) holds, given the threshold pair $(\bar{c}, g(\bar{c}))$. We examine the range of $B(\bar{c})$ for $\bar{c} \in (c_L, c_0]$. If $c_0 < c_H$, we observe the following:

- When $\bar{c} \to c_L^+$, we have $g(\bar{c}) \to \tilde{c}$, and hence, $B(\bar{c}) \to \left[\frac{1}{c_L} - \int_{c_L}^{\tilde{c}} \frac{[1-F(y)]^{N-1}}{y^2}\, dy\right]^{-1} = \underline{B}$.
- When $\bar{c} \to c_0^-$, since $w(c_0) = 0$, we have $g(\bar{c}) \to c_0$, and hence, $B(\bar{c}) \to \frac{N c_0 F(c_0)}{1 - [1 - F(c_0)]^N} = \bar{B}$.

By continuity, for any budget level $\underline{B} < B < \bar{B}$, there exists a threshold pair $(\bar{c}, \check{c})$ that satisfies the system of equations (20).

When $\tilde{c} < c_0 = c_H$, we observe the following:

- When $\bar{c} \to c_L^+$, we have $g(\bar{c}) \to \tilde{c}$, and hence, $B(\bar{c}) \to \left[\frac{1}{c_L} - \int_{c_L}^{\tilde{c}} \frac{[1-F(y)]^{N-1}}{y^2}\, dy\right]^{-1} = \underline{B}$.
- When $\bar{c} = \tilde{c}_1$, we have $g(\bar{c}) = c_H$, and hence, $B(\bar{c}) \to \tilde{B}$.

By continuity, for any budget level $\underline{B} < B < \tilde{B}$, there exists a threshold pair $(\bar{c}, \check{c})$ that satisfies the system of equations (20). Combining the arguments above establishes the existence of a threshold pair $(\bar{c}, \check{c})$ under the given conditions.

Next, we will verify the feasibility of Q and M, given the existence of $(\bar{c}, \check{c})$. In the proof of Proposition 1, we have established the monotonicity of Q on $[\bar{c}, \check{c}]$. The desired feasibility condition for Q boils down to

$$Q(\bar{c}) \leq 1 \quad \Leftrightarrow \quad \frac{[1-F(\bar{c})]^{N-1}}{\bar{c}} - \int_{\bar{c}}^{\check{c}} \frac{[1-F(y)]^{N-1}}{y^2} \, \mathrm{d}y \leq \frac{1}{B}$$

and

$$Q(\check{c}) \geq 0 \quad \Leftrightarrow \quad \frac{B\,[1-F(\check{c})]^{N-1}}{\check{c}} \geq 0,$$

where the second inequality holds automatically. To prove the first inequality, because

$$\int_{\bar{c}}^{\check{c}} \frac{[1-F(y)]^{N-1}}{y^2} \, \mathrm{d}y = \frac{1}{N\,\bar{c}\,F(\bar{c})} \left[1 - [1-F(\bar{c})]^N\right] - \frac{1}{B},$$

and hence, the desired feasibility condition can be rewritten as

$$N\,F(\bar{c})\,[1-F(\bar{c})]^{N-1} + [1-F(\bar{c})]^N - 1 \leq 0.$$

Define $g(x) \equiv N\,x\,(1-x)^{N-1} + (1-x)^N$. Taking derivative yields

$$g'(x) = -(N-1)\,x\,(1-x)^{N-2} < 0.$$

Because $g(0) = 1$, we know that the desired nonnegativity condition is implied by the existence of threshold $\bar{c}$.

By construction, we have

$$\int_{c_L}^{c} M(y)\,\mathrm{d}y = \frac{B}{N} \left[1 - [1-F(c)]^N\right]$$

for any $c \in [\bar{c}, \check{c}]$. Moreover, since M is constant on $[c_L, \bar{c}]$, the Border's constraint in (9) continues to hold.

This concludes the proof of primal feasibility. It remains to establish the optimality of our candidate solution. Let

$$\begin{aligned}
\sigma(c) &= w(c)\,f(c) - \frac{[F(c) + c\,f(c)]\,w(\bar{c})\,f(\bar{c})}{F(\bar{c}) + \bar{c}\,f(\bar{c})}, && \forall c \in [c_L, \bar{c}], \\
\xi(c) &= -\frac{\mathrm{d}}{\mathrm{d}c}\left[\frac{1}{c^2\,f(c)}\left[c\,w(c)\,f(c) - \check{c}\,w(\check{c})\,f(\check{c}) + \int_c^{\check{c}} w(y)\,f(y)\,\mathrm{d}y\right]\right], && \forall c \in [\bar{c}, \check{c}], \\
\delta(c) &= -w(c)\,f(c) + w(\check{c})\,f(\check{c}), && \forall c \in [\check{c}, c_H].
\end{aligned}$$

With a slight abuse of notation, we continue to use J to denote the function

$$\frac{1}{c^2 f(c)}\left[c\, w(c)\, f(c) - \check{c}\, w(\check{c})\, f(\check{c}) + \int_c^{\check{c}} w(y)\, f(y)\, \mathrm{d}y\right].$$

Differentiating σ yields

$$\begin{aligned}
\sigma'(c) &= (R-c)\, f'(c) - 2\, f(c) - \frac{w(\bar{c})\, f(\bar{c})}{F(\bar{c}) + \bar{c}\, f(\bar{c})}\, [c\, f'(c) + 2\, f(c)] \\
&= R\, f'(c) - \left[\frac{w(\bar{c})\, f(\bar{c})}{F(\bar{c}) + \bar{c}\, f(\bar{c})} + 1\right] [c\, f'(c) + 2\, f(c)] \\
&= R\left[f'(c) - \frac{f(\bar{c})\, [c\, f'(c) + 2\, f(c)]}{F(\bar{c}) + \bar{c}\, f(\bar{c})}\right] \\
&= \frac{f'(c)\, [F(\bar{c}) + f(\bar{c})\, (\bar{c} - c)] - 2\, f(c)\, f(\bar{c})}{F(\bar{c}) + \bar{c}\, f(\bar{c})} \\
&< 0,
\end{aligned}$$

where the inequality is implied by Assumption 1. Therefore, the desired nonnegativity condition reduces to $\sigma(\bar{c}) \geq 0$, which follows directly from its definition. The nonnegativity of ξ is equivalent to the monotonicity of J, which can be established using the same argument as in the proof of Proposition 1. Lastly, the nonnegativity of δ is ensured by the monotonicity of the function $w(c)\, f(c)$.

Because

$$\begin{aligned}
&\sigma(c) + \int_{c_L}^{c}\left(\int_{\bar{c}}^{\check{c}} \xi(z)\, \mathrm{d}z\right) f(y)\, \mathrm{d}y + c\, f(c) \int_{\bar{c}}^{\check{c}} \xi(y)\, \mathrm{d}y = w(c)\, f(c), && \forall c \in [c_L, \bar{c}], \\
&\int_{c_L}^{\bar{c}}\left(\int_{\bar{c}}^{\check{c}} \xi(z)\, \mathrm{d}z\right) f(y)\, \mathrm{d}y + \int_{\bar{c}}^{c}\left(\int_{y}^{\check{c}} \xi(z)\, \mathrm{d}z\right) f(y)\, \mathrm{d}y + c\, f(c) \int_{c}^{\check{c}} \xi(y)\, \mathrm{d}y = w(c)\, f(c), && \forall c \in [\bar{c}, \check{c}], \text{ and} \\
&-\delta(c) + \int_{c_L}^{\bar{c}}\left(\int_{\bar{c}}^{\check{c}} \xi(z)\, \mathrm{d}z\right) f(y)\, \mathrm{d}y + \int_{\bar{c}}^{\check{c}}\left(\int_{y}^{\check{c}} \xi(z)\, \mathrm{d}z\right) f(y)\, \mathrm{d}y = w(c)\, f(c), && \forall c \in [\check{c}, c_H],
\end{aligned}$$

we have

$$\begin{aligned}
&\int_{c_L}^{\bar{c}} Q(c)\, \sigma(c)\, \mathrm{d}c + \int_{\bar{c}}^{\check{c}}\left[\int_{c_L}^{c}\left[\int_{y}^{c_H} Q(z)\, \mathrm{d}z + y\, Q(y)\right] \mathrm{d}F(y)\right] \xi(c)\, \mathrm{d}c + \int_{\check{c}}^{c_H} [-Q(c)]\, \delta(c)\, \mathrm{d}c \\
= &\int_{c_L}^{\bar{c}}\left[\sigma(c) + \int_{c_L}^{c}\left(\int_{\bar{c}}^{\check{c}} \xi(z)\, \mathrm{d}z\right) f(y)\, \mathrm{d}y + c\, f(c) \int_{\bar{c}}^{\check{c}} \xi(y)\, \mathrm{d}y\right] Q(c)\, \mathrm{d}c \\
&+ \int_{\bar{c}}^{\check{c}}\left[\int_{c_L}^{\bar{c}}\left(\int_{\bar{c}}^{\check{c}} \xi(z)\, \mathrm{d}z\right) f(y)\, \mathrm{d}y + \int_{\bar{c}}^{c}\left(\int_{y}^{\check{c}} \xi(z)\, \mathrm{d}z\right) f(y)\, \mathrm{d}y + c\, f(c) \int_{c}^{\check{c}} \xi(y)\, \mathrm{d}y\right] Q(c)\, \mathrm{d}c \\
&+ \int_{\check{c}}^{c_H}\left[-\delta(c) + \int_{c_L}^{\bar{c}}\left(\int_{\bar{c}}^{\check{c}} \xi(z)\, \mathrm{d}z\right) f(y)\, \mathrm{d}y + \int_{\bar{c}}^{\check{c}}\left(\int_{y}^{\check{c}} \xi(z)\, \mathrm{d}z\right) f(y)\, \mathrm{d}y\right] Q(c)\, \mathrm{d}c \\
= &\int_{c_L}^{c_H} w(c)\, Q(c)\, f(c),
\end{aligned}$$

and hence, the objective function is bounded above by

$$\int_{c_L}^{\bar{c}} \sigma(c)\, \mathrm{d}c + \frac{B}{N} \int_{\bar{c}}^{\check{c}} \left[1 - [1 - F(c)]^N\right] \xi(c)\, \mathrm{d}c.$$

Our candidate solution achieves this upper bound, as shown by

$$
\begin{aligned}
&\int_{c_L}^{\bar{c}} \sigma(c)\,\mathrm{d}c + \frac{B}{N}\int_{\bar{c}}^{\check{c}} \xi(c)\,\left[1-[1-F(c)]^N\right]\mathrm{d}c - \int_{c_L}^{\bar{c}} w(c)\,\mathrm{d}F(c) - \int_{\bar{c}}^{\check{c}} w(c)\,Q(c)\,\mathrm{d}F(c)\\
={}& \int_{c_L}^{\bar{c}} \left[w(c)\,f(c) - \frac{[F(c)+c\,f(c)]\;w(\bar{c})\,f(\bar{c})}{F(\bar{c})+\bar{c}\,f(\bar{c})}\right]\mathrm{d}c - \frac{B}{N}\int_{\bar{c}}^{\check{c}}\left[1-[1-F(c)]^N\right]\mathrm{d}J(c)\\
&-\int_{c_L}^{\bar{c}} w(c)\,\mathrm{d}F(c) - B\int_{\bar{c}}^{\check{c}} w(c)\,\left[\frac{[1-F(c)]^{N-1}}{c} - \int_c^{\check{c}}\frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y\right]\mathrm{d}F(c)\\
={}& -\left[\bar{c}\,F(\bar{c}) - \underbrace{\frac{B}{N}\left[1-[1-F(\bar{c})]^N\right]}_{=\bar{c}\left[B\int_{\bar{c}}^{\check{c}}\frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y+1\right]F(\bar{c})}\right]\underbrace{J(\bar{c})}_{=\frac{w(\bar{c})\,f(\bar{c})}{F(\bar{c})+\bar{c}\,f(\bar{c})}}\\
&+B\int_{\bar{c}}^{\check{c}}\frac{1}{c^2}\left[c\,w(c)\,f(c) - \check{c}\,w(\check{c})\,f(\check{c}) + \int_c^{\check{c}} w(y)\,f(y)\,\mathrm{d}y\right](1-F(c))^{N-1}\,\mathrm{d}c\\
&-B\int_{\bar{c}}^{\check{c}} w(c)\,\left[\frac{[1-F(c)]^{N-1}}{c} - \int_c^{\check{c}}\frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y\right]\mathrm{d}F(c)\\
={}& \left[B\,\bar{c}\int_{\bar{c}}^{\check{c}}\frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y\,F(\bar{c})\right]\frac{w(\bar{c})\,f(\bar{c})}{F(\bar{c})+\bar{c}\,f(\bar{c})}\\
&+B\int_{\bar{c}}^{\check{c}}\frac{1}{c^2}\left[c\,w(c)\,f(c) - \check{c}\,w(\check{c})\,f(\check{c}) + \int_c^{\check{c}} w(y)\,f(y)\,\mathrm{d}y\right](1-F(c))^{N-1}\,\mathrm{d}c\\
&-B\int_{\bar{c}}^{\check{c}} w(c)\,\left[\frac{[1-F(c)]^{N-1}}{c} - \int_c^{\check{c}}\frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y\right]\mathrm{d}F(c)\\
={}& B\int_{\bar{c}}^{\check{c}}\left[\frac{\bar{c}\,w(\bar{c})\,F(\bar{c})\,f(\bar{c})}{F(\bar{c})+\bar{c}\,f(\bar{c})} - \check{c}\,w(\check{c})\,f(\check{c}) + \int_{\bar{c}}^{\check{c}} w(y)\,f(y)\,\mathrm{d}y\right]\frac{[1-F(c)]^{N-1}}{c^2}\,\mathrm{d}c\\
={}& 0,
\end{aligned}
$$

where the last step follows from the first equation in (20) that determines $\bar{c}$ and $\check{c}$.

This completes the proof. □

Proof of Corollary 2 By construction, for any $c_1 \in [c_L, c_H]$, we have

$$\int_{c_{-i}\in[c_L,c_H]^{N-1}} q(c_i, c_{-i}) \prod_{k\in[N]\setminus\{i\}} \mathrm{d}F(c_k) = Q(c_i).$$

For any $c \in [c_L, \bar{c}]$, we have

$$
\begin{aligned}
\int_{c_{-i}\in[c_L,c_H]^{N-1}} m(c_i, c_{-i}) \prod_{k\in[N]\setminus\{i\}} \mathrm{d}F(c_k) &= \sum_{j=0}^{N-1}\frac{B}{j+1}\binom{N-1}{j} F(\bar{c})^j\,[1-F(\bar{c})]^{N-1-j}\\
&= \sum_{j=0}^{N-1}\frac{B}{j+1}\,\frac{(N-1)!}{j!\,(N-j-1)!}\,F(\bar{c})^j\,[1-F(\bar{c})]^{N-1-j}\\
&= \frac{B}{N\,F(\bar{c})}\sum_{j=0}^{N-1}\frac{N!}{(j+1)!\,(N-j-1)!}\,F(\bar{c})^{j+1}\,[1-F(\bar{c})]^{N-1-j}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{B}{N\,F(\bar{c})} \sum_{k=1}^{N} \binom{N}{k} F(\bar{c})^k \, [1-F(\bar{c})]^{N-k} \\
&= \frac{B}{N\,F(\bar{c})} \left[1-[1-F(\bar{c})]^N\right] \\
&= \bar{c}\left[B \int_{\bar{c}}^{\check{c}} \frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y + 1\right],
\end{aligned}
$$

where the last equality follows from the second equation in (20).

For any $c_1 \in [\bar{c}, \check{c}]$, we have

$$
M(c) = B \cdot \Pr(\text{all its } N-1\text{'s competitors' types are below } c) = B\left[1-[1-F(c)]^{N-1}\right].
$$

Meanwhile, for any reporting profile $(c_i, c_{-i}) \in [c_L, c_H]^N$, we have $\sum_{i=1}^{N} m(c_i, c_{-i}) \leq B$ and $0 \leq q(c_i, c_{-i}) \leq 1$, implying its feasibility.

This completes the proof. $\square$

Proof of Proposition 3 We first verify the existence of $\bar{c}$ under the given conditions. Let

$$
B(\bar{c}) = \frac{\bar{c}\,F(\bar{c})}{\frac{1}{N}[1-[1-F(\bar{c})]^N] - \bar{c}\,F(\bar{c}) \int_{\bar{c}}^{c_H} \frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y},
$$

which represents the unique budget level at which equation (24) holds for a given threshold $\bar{c}$. Note that $B(\bar{c})$ is increasing in $\bar{c}$, which follows from

$$
\begin{aligned}
B(\cdot) \text{ increasing} \quad &\Leftrightarrow \quad \frac{1-[1-F(c)]^N}{N\,c\,F(c)} - \int_{c}^{c_H} \frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y \text{ decreasing} \\
&\Leftrightarrow \quad N\,F(c)\,[1-F(c)]^{N-1} + [1-F(c)]^N - 1 \leq 0.
\end{aligned}
$$

Since $N\,F(c)\,[1-F(c)]^{N-1} + [1-F(c)]^N - 1$ is decreasing, the desired property is reduced to

$$
\left[N\,F(c)\,[1-F(c)]^{N-1} + [1-F(c)]^N - 1\right]_{c=c_L} \leq 0,
$$

which holds automatically from its definition. Therefore, we conclude that $\bar{c}$ is increasing in B.

When $\tilde{c} < c_0 = c_H$, we have $B(\tilde{c}_1) = \tilde{B}$ and $B(c_H) = N\,c_H$. The continuity property implies that for any $\tilde{B} < B \leq \bar{B}$, there exists $\bar{c} \in [\tilde{c}_1, c_H]$ satisfying (24). Similarly, when $\tilde{c} = c_H$ and $\underline{B} < B \leq \bar{B} = N\,c_H$, we have $\lim_{c \downarrow c_L} B(c) = \underline{B}$ and $B(c_H) = N\,c_H$. The same argument also holds in this case.

Next, we establish the feasibility of our candidate solution, assuming the existence of $\bar{c}$. The monotonicity of Q and its feasibility—specifically, $Q(\bar{c}) \leq 1$ and $Q(c_H) \geq 0$—follow from the same argument used in the proof of Proposition 2. As for M, since the Border's constraint (9) is binding for all $c \in [\bar{c}, c_H]$, it continues to hold on the interval $[c_L, \bar{c}]$ as well.

Let

$$\begin{aligned}
&\sigma(c) = w(c)\,f(c) - \frac{[F(c)+c\,f(c)]\;w(\bar c)\,f(\bar c)}{F(\bar c)+\bar c\,f(\bar c)}, && \forall c \in [c_L, \bar c],\\
&\xi(c) = -\phi'(c), && \forall c \in [\bar c, c_H],\\
&\xi_T = \phi(c_H),
\end{aligned}$$

where

$$\phi(c) = \frac{1}{c^2\,f(c)}\underbrace{\left[c\,w(c)\,f(c) - \int_{\bar c}^{c} w(y)\,f(y)\,\mathrm{d}y - \frac{\bar c\,w(\bar c)\,F(\bar c)\,f(\bar c)}{F(\bar c)+\bar c\,f(\bar c)}\right]}_{\equiv N(c)}.$$

Since

$$\begin{aligned}
\xi_T \ge 0 \quad &\Leftrightarrow \quad N(c_H) \ge 0\\
&\Leftrightarrow \quad c_H\,w(c_H)\,f(c_H) - \int_{\bar c}^{c_H} w(y)\,f(y)\,\mathrm{d}y - \frac{\bar c\,w(\bar c)\,F(\bar c)\,f(\bar c)}{F(\bar c)+\bar c\,f(\bar c)} \ge 0\\
&\Leftrightarrow \quad -h(c_H) + h(\bar c) + \frac{\bar c^2\,w(\bar c)\,f(\bar c)^2}{F(\bar c)+\bar c\,f(\bar c)} \ge 0,
\end{aligned}$$

where h is defined by (EC.9), to verify the nonnegativity of ξ_T, it suffices to show that

$$-h(c_H) + h(\bar c) + \frac{\bar c^2\,w(\bar c)\,f(\bar c)^2}{F(\bar c)+\bar c\,f(\bar c)} \ge 0.$$

Recall that the monotonicity of $h(c) + \frac{c^2\,w(c)\,f(c)^2}{F(c)+c\,f(c)}$ was established in the proof of Proposition 2. When $\tilde c < c_0 = c_H$ and $B = \tilde B$, the inequality above holds with equality. Since $\bar c$ is increasing in B, the desired result holds automatically.

When $\tilde c = c_H$, it suffices to show that

$$-h(c_H) + h(c_L) + c_L\,w(c_L)\,f(c_L) \ge 0,$$

which follows from

$$-h(c_H) + h(c_L) + c_L\,w(c_L)\,f(c_L) = -h(c_H) \ge 0.$$

The last inequality follows from the definition of $\tilde c$.

The nonnegativity of ξ is equivalent to the monotonicity of ϕ. The desired result boils down to

$$c^2\,f(c)\underbrace{N'(c)}_{=c\,\frac{\mathrm{d}}{\mathrm{d}c}(w(c)\,f(c))\le 0} - c\,N(c)\;[2\,f(c) + c\,f'(c)] \le 0.$$

Notice that

$$\xi_T \ge 0 \quad \Leftrightarrow \quad N(c_H) \ge 0.$$

Hence, a sufficient condition is given by $2\,f(c)+c\,f'(c)\geq 0$. It remains to prove the inequality when $2\,f(c)+c\,f'(c)<0$. The expression of ϕ can be rewritten as

$$\phi(c)=\frac{1}{c^2\,f(c)}\left[\underbrace{c\,w(c)\,f(c)-(R-c)\,F(c)}_{=c\,(R-c)\,f(c)-R\,F(c)}+\underbrace{(R-\bar{c})\,F(\bar{c})-\frac{\bar{c}\,w(\bar{c})\,F(\bar{c})\,f(\bar{c})}{F(\bar{c})+\bar{c}\,f(\bar{c})}}_{=\frac{R\,F(\bar{c})^2}{F(\bar{c})+\bar{c}\,f(\bar{c})}}\right],$$

and we have

$$\phi'(c)=\frac{-2\,R\,c^2\,f(c)^2+R\left[F(c)-\frac{F(\bar{c})^2}{F(\bar{c})+\bar{c}\,f(\bar{c})}\right][2\,c\,f(c)+c^2\,f'(c)]}{c^4\,f(c)^2}.$$

The corresponding monotonicity condition is given by

$$-2\,R\,c^2\,f(c)^2+R\underbrace{\left[F(c)-\frac{F\left(\bar{c}\right)^2}{F\left(\bar{c}\right)+\bar{c}\,f\left(\bar{c}\right)}\right]}_{\geq F(c)-F(\bar{c})\geq 0}\left[2\,c\,f(c)+c^2\,f'(c)\right]\leq 0,$$

where the inequality is implied by $2\,f(c)+c\,f'(c)<0$.

The nonnegativity of σ follows from

$$\sigma(c)=w(c)\,f(c)-\frac{w(\bar{c})\,f(\bar{c})\,\left[F(c)+c\,f(c)\right]}{F(\bar{c})+\bar{c}\,f(\bar{c})}=R\left[F(\bar{c})+\bar{c}\,f(\bar{c})-\left[c+\frac{F(c)}{f(c)}\right]\,f(\bar{c})\right]\,f(c)\geq 0,$$

where the inequality follows from the monotonicity of $c+\frac{F(c)}{f(c)}$.

Because

$$\sigma(c)+\left[F(c)+c\,f(c)\right]\left[\int_{\bar{c}}^{c_H}\xi(y)\,\mathrm{d}y+\xi_T\right]=w(c)\,f(c),\qquad\forall c\in[c_L,\bar{c}],$$

$$\left[\int_{\bar{c}}^{c_H}\xi(z)\,\mathrm{d}z+\xi_T\right]F(\bar{c})+\int_{\bar{c}}^{c}\left[\int_{y}^{c_H}\xi(z)\,\mathrm{d}z+\xi_T\right]f(y)\,\mathrm{d}y+c\,f(c)\left[\int_{c}^{c_H}\xi(y)\,\mathrm{d}y+\xi_T\right]=w(c)\,f(c),\quad\forall c\in[\bar{c},c_H],$$

we have

$$\begin{aligned}
&\int_{c_L}^{\bar{c}}Q(c)\,\sigma(c)\,\mathrm{d}c+\int_{\bar{c}}^{c_H}\left[\int_{c_L}^{c}\left[\int_{y}^{c_H}Q(z)\,\mathrm{d}z+y\,Q(y)\right]\mathrm{d}F(y)\right]\xi(c)\,\mathrm{d}c+\xi_T\int_{c_L}^{c_H}\left[\int_{y}^{c_H}Q(z)\,\mathrm{d}z+y\,Q(y)\right]\mathrm{d}F(y)\\
=&\int_{c_L}^{\bar{c}}\left[\sigma(c)+\left[F(c)+c\,f(c)\right]\left[\int_{\bar{c}}^{c_H}\xi(y)\,\mathrm{d}y+\xi_T\right]\right]Q(c)\,\mathrm{d}c\\
&+\int_{\bar{c}}^{c_H}\left[\left[\int_{\bar{c}}^{c_H}\xi(z)\,\mathrm{d}z+\xi_T\right]F(\bar{c})+\int_{\bar{c}}^{c}\left[\int_{y}^{c_H}\xi(z)\,\mathrm{d}z+\xi_T\right]f(y)\,\mathrm{d}y+c\,f(c)\left[\int_{c}^{c_H}\xi(y)\,\mathrm{d}y+\xi_T\right]\right]Q(c)\,\mathrm{d}c\\
=&\int_{c_L}^{c_H}w(c)\,Q(c)\,f(c).
\end{aligned}$$

Hence, the objective function is bounded above by

$$\int_{c_L}^{\bar{c}}\sigma(c)\,\mathrm{d}c+\frac{B}{N}\int_{\bar{c}}^{c_H}\left[1-\left[1-F(c)\right]^N\right]\xi(c)\,\mathrm{d}c+\frac{B}{N}\,\xi_T.$$

Our candidate solution achieves this upper bound, as shown by

$$
\begin{aligned}
&\int_{c_L}^{\bar c} \sigma(c)\,\mathrm{d}c + \frac{B}{N}\int_{\bar c}^{c_H} \xi(c)\,\left[1-[1-F(c)]^N\right]\mathrm{d}c + \frac{B}{N}\,\xi_T - \int_{c_L}^{\bar c} w(c)\,\mathrm{d}F(c) - \int_{\bar c}^{c_H} w(c)\,Q(c)\,\mathrm{d}F(c)\\
=&\int_{c_L}^{\bar c}\left[w(c)\,f(c) - \frac{[F(c)+c\,f(c)]\;w(\bar c)\,f(\bar c)}{F(\bar c)+\bar c\,f(\bar c)}\right]\mathrm{d}c - \frac{B}{N}\int_{\bar c}^{c_H}\left[1-[1-F(c)]^N\right]\mathrm{d}\phi(c)\\
&+\frac{B}{N}\,\frac{1}{c_H^2\,f(c_H)}\left[c_H\,w(c_H)\,f(c_H) - \int_{\bar c}^{c_H} w(y)\,f(y)\,\mathrm{d}y - \frac{\bar c\,w(\bar c)\,F(\bar c)\,f(\bar c)}{F(\bar c)+\bar c\,f(\bar c)}\right] - \int_{c_L}^{\bar c} w(c)\,\mathrm{d}F(c)\\
&-B\int_{\bar c}^{c_H} w(c)\left[\frac{[1-F(c)]^{N-1}}{c} - \int_c^{c_H}\frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y\right]\mathrm{d}F(c)\\
=&-\int_{c_L}^{\bar c}\frac{[F(c)+c\,f(c)]\;w(\bar c)\,f(\bar c)}{F(\bar c)+\bar c\,f(\bar c)}\,\mathrm{d}c\\
&-\frac{B}{N}\left[\phi(c_H) - \left[1-[1-F(\bar c)]^N\right]\phi(\bar c) - \int_{\bar c}^{c_H}\phi(c)\,\mathrm{d}\left[1-[1-F(c)]^N\right]\right]\\
&+\frac{B}{N}\,\frac{1}{c_H^2\,f(c_H)}\left[c_H\,w(c_H)\,f(c_H) - \int_{\bar c}^{c_H} w(y)\,f(y)\,\mathrm{d}y - \frac{\bar c\,w(\bar c)\,F(\bar c)\,f(\bar c)}{F(\bar c)+\bar c\,f(\bar c)}\right]\\
&-B\int_{\bar c}^{c_H} w(c)\left[\frac{[1-F(c)]^{N-1}}{c} - \int_c^{c_H}\frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y\right]\mathrm{d}F(c)\\
=&-\frac{\bar c\,w(\bar c)\,F(\bar c)\,f(\bar c)}{F(\bar c)+\bar c\,f(\bar c)} + \frac{B}{N}\left[1-[1-F(\bar c)]^N\right]\frac{w(\bar c)\,f(\bar c)}{F(\bar c)+\bar c\,f(\bar c)}\\
&+B\int_{\bar c}^{c_H}\frac{1}{c^2}\left[c\,w(c)\,f(c) - \int_{\bar c}^{c} w(y)\,f(y)\,\mathrm{d}y - \frac{\bar c\,w(\bar c)\,F(\bar c)\,f(\bar c)}{F(\bar c)+\bar c\,f(\bar c)}\right][1-F(c)]^{N-1}\,\mathrm{d}c\\
&-B\int_{\bar c}^{c_H} w(c)\left[\frac{[1-F(c)]^{N-1}}{c} - \int_c^{c_H}\frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y\right]\mathrm{d}F(c)\\
=&-\frac{\bar c\,w(\bar c)\,F(\bar c)\,f(\bar c)}{F(\bar c)+\bar c\,f(\bar c)} + \underbrace{\frac{B}{N}\left[1-[1-F(\bar c)]^N\right]}_{=\bar c\left[B\int_{\bar c}^{c_H}\frac{[1-F(y)]^{N-1}}{y^2}\,\mathrm{d}y+1\right]F(\bar c)}\frac{w(\bar c)\,f(\bar c)}{F(\bar c)+\bar c\,f(\bar c)}\\
&-B\,\frac{\bar c\,w(\bar c)\,F(\bar c)\,f(\bar c)}{F(\bar c)+\bar c\,f(\bar c)}\int_{\bar c}^{c_H}\frac{1}{c^2}\,[1-F(c)]^{N-1}\,\mathrm{d}c\\
=&\;0.
\end{aligned}
$$

This completes the proof. □

Proof of Corollary 3 The proof follows analogously to that of Corollary 2 and is therefore omitted. □

Proof of Proposition 4 Consider a relaxed problem that ignores all constraints except the feasibility condition on Q. Given that the virtual valuation function w is decreasing, the optimal solution to this problem is $Q(c)=\mathbb{1}_{c\le c_0}$. Under this Q, the corresponding M recovered from envelope condition (EC.4) is given by $M(c)=c_0\cdot\mathbb{1}_{c\le c_0}$.

For any $c\in[c_L,c_0]$, we have

$$\int_{c_L}^{c} M(y)\,\mathrm{d}F(y) = c_0\,F(c),$$

and the corresponding feasibility condition is given by

$$c_0\, F(c) \le \frac{B}{N}\, \left[1-[1-F(c)]^N\right] \quad \Leftrightarrow \quad B \ge \frac{N\, c_0\, F(c_0)}{1-[1-F(c_0)]^N},$$

indicating that the solution to the relaxed problem remains feasible when the previously ignored constraints are reintroduced.

This establishes the optimality of the solution and thus completes the proof. □

Proof of Corollary 4 By construction, for any $c_1 \in [c_L, c_H]$, we have

$$\int_{c_{-i}\in[c_L,c_H]^{N-1}} q(c_i, c_{-i}) \prod_{k\in[N]\setminus\{i\}} \mathrm{d}F(c_k) = Q(c_i).$$

For any $c \in [c_L, c_0]$, we have

$$\begin{aligned}
\int_{c_{-i}\in[c_L,c_H]^{N-1}} m(c_i, c_{-i}) \prod_{k\in[N]\setminus\{i\}} \mathrm{d}F(c_k) &= \sum_{j=0}^{N-1} \frac{\bar{B}}{j+1} \binom{N-1}{j} F(c_0)^j\, [1-F(c_0)]^{N-1-j} \\
&= \sum_{j=0}^{N-1} \frac{\bar{B}}{j+1}\, \frac{(N-1)!}{j!\,(N-j-1)!}\, F(c_0)^j\, [1-F(c_0)]^{N-1-j} \\
&= \frac{\bar{B}}{N\, F(c_0)} \sum_{j=0}^{N-1} \frac{N!}{(j+1)!\,(N-j-1)!}\, F(c_0)^{j+1}\, [1-F(c_0)]^{N-1-j} \\
&= \frac{\bar{B}}{N\, F(c_0)} \sum_{k=1}^{N} \binom{N}{k} F(c_0)^k\, [1-F(c_0)]^{N-k} \\
&= \frac{\bar{B}}{N\, F(c_0)} \left[1-[1-F(c_0)]^N\right] \\
&= c_0.
\end{aligned}$$

For any $c \in [c_0, c_H]$, we have

$$\int_{c_{-i}\in[c_L,c_H]^{N-1}} m(c_i, c_{-i}) \prod_{k\in[N]\setminus\{i\}} \mathrm{d}F(c_k) = 0,$$

confirming that taking expectation of $m(c_i, c_{-i})$ with respect to c_{-i} yields $M(c)$.

Meanwhile, for any reporting profile $(c_i, c_{-i}) \in [c_L, c_H]^N$, we have $\sum_{i=1}^N m(c_i, c_{-i}) \le B$ and $0 \le q(c_i, c_{-i}) \le 1$, implying its feasibility.

This completes the proof. □